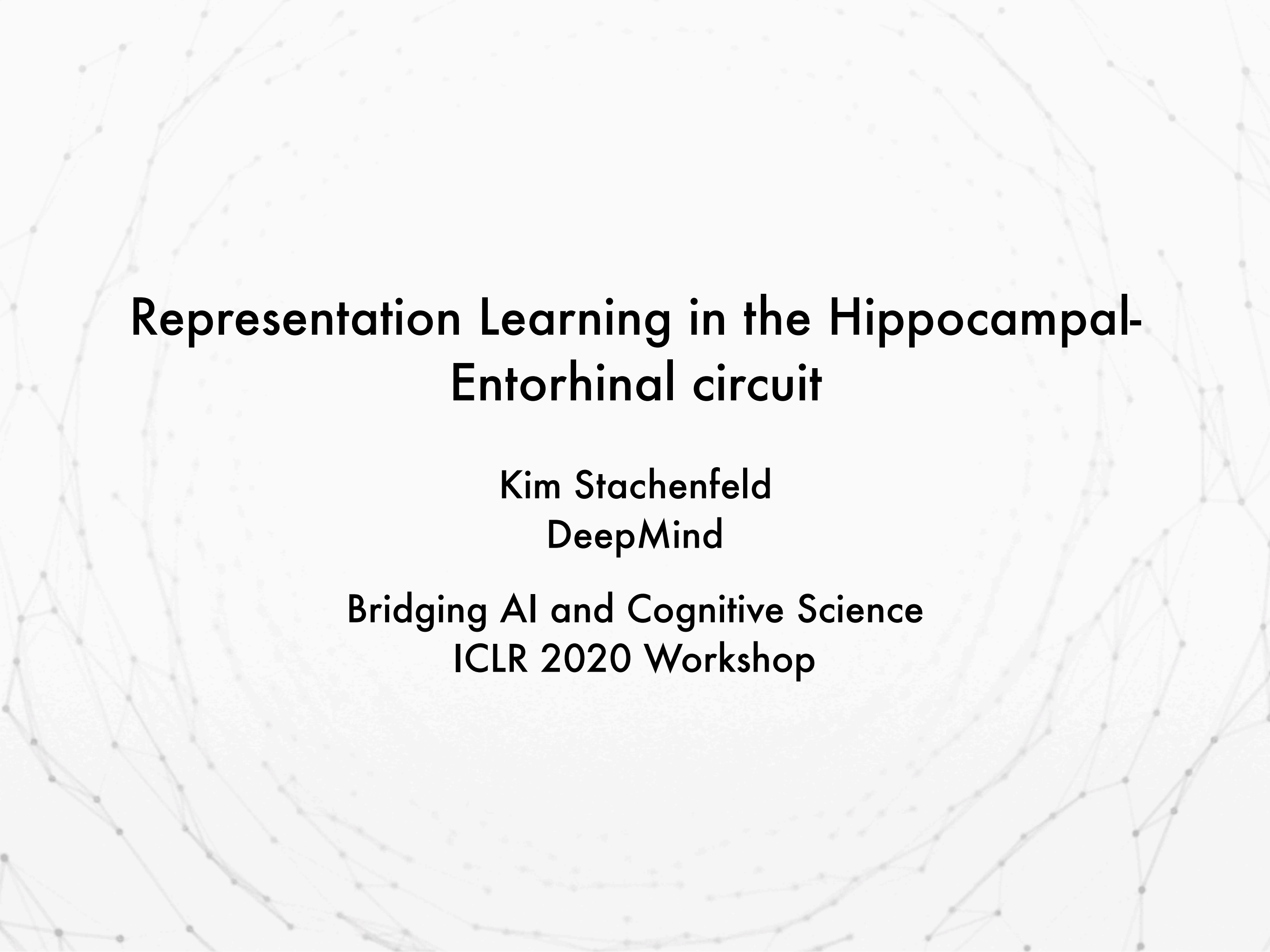




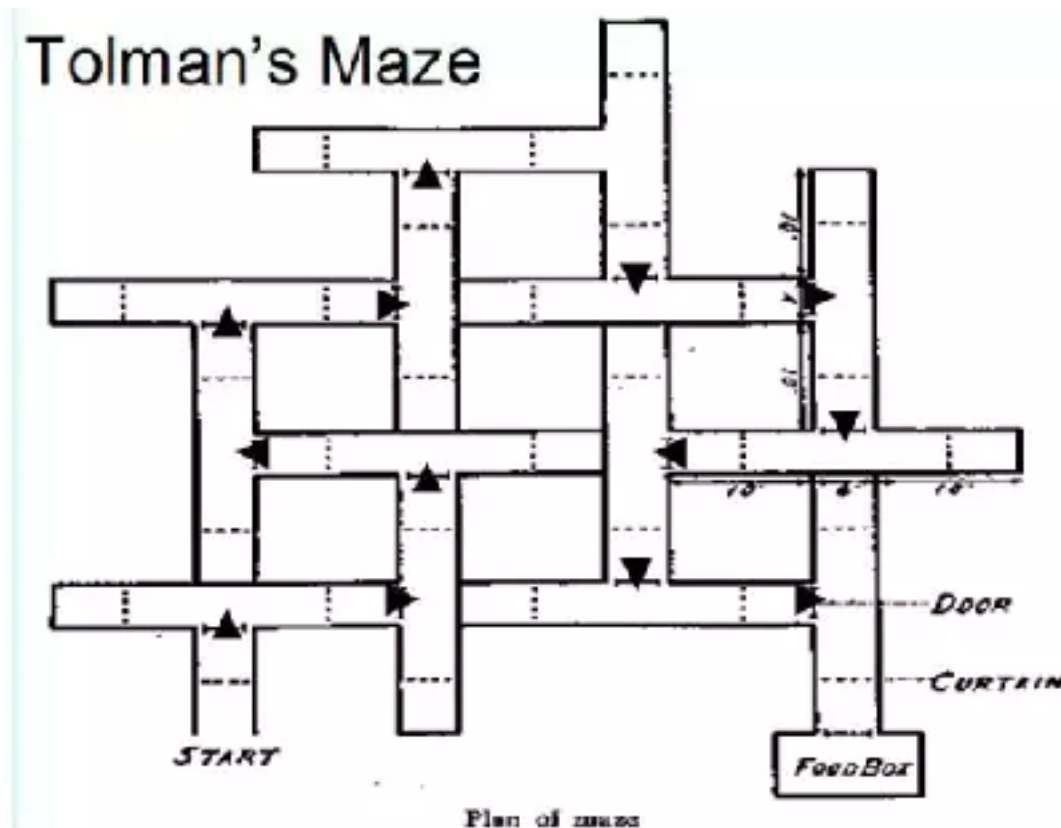
# **Representation Learning in the Hippocampal-Entorhinal circuit**

**Kim Stachenfeld  
DeepMind**

**Bridging AI and Cognitive Science  
ICLR 2020 Workshop**



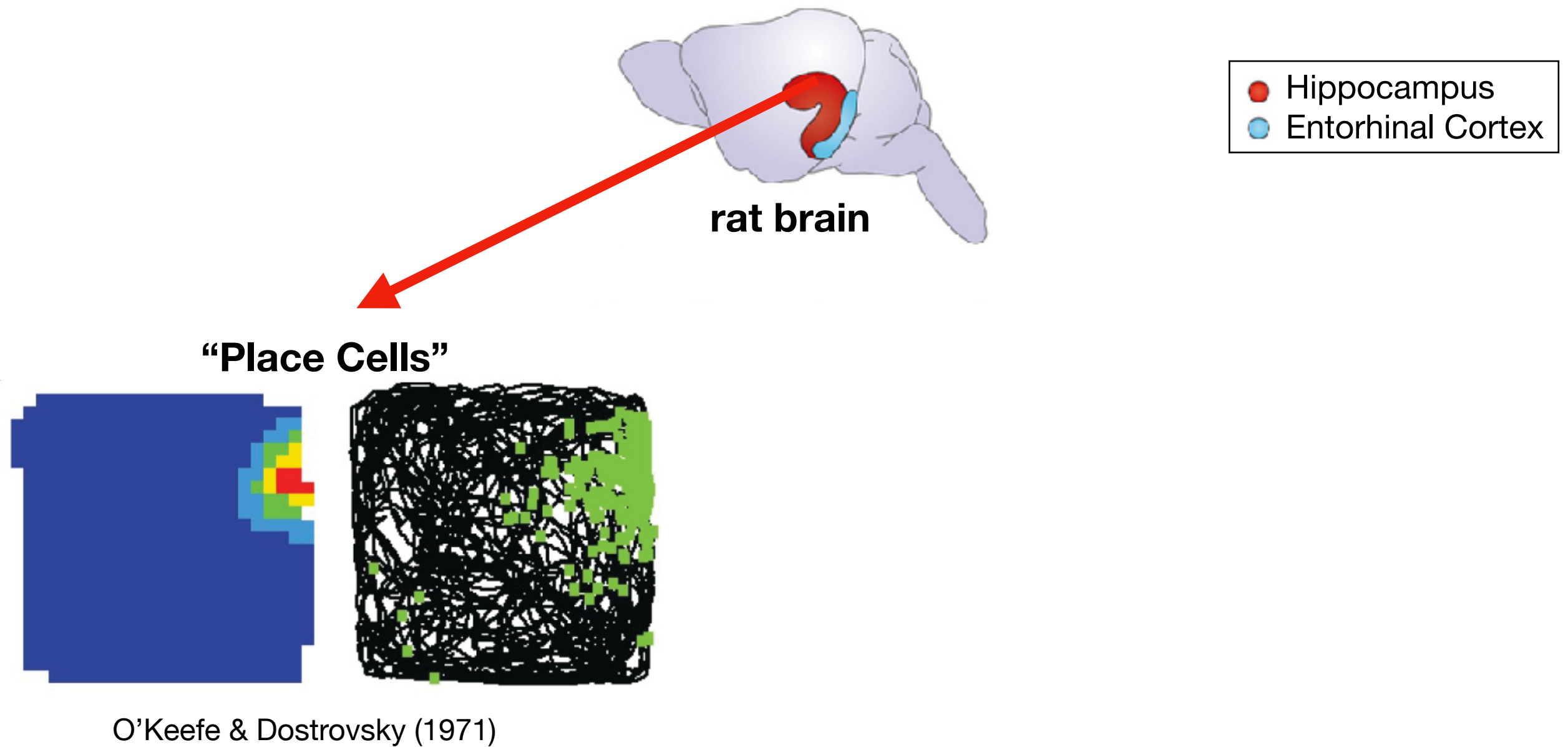
# Tolman and the Cognitive Map



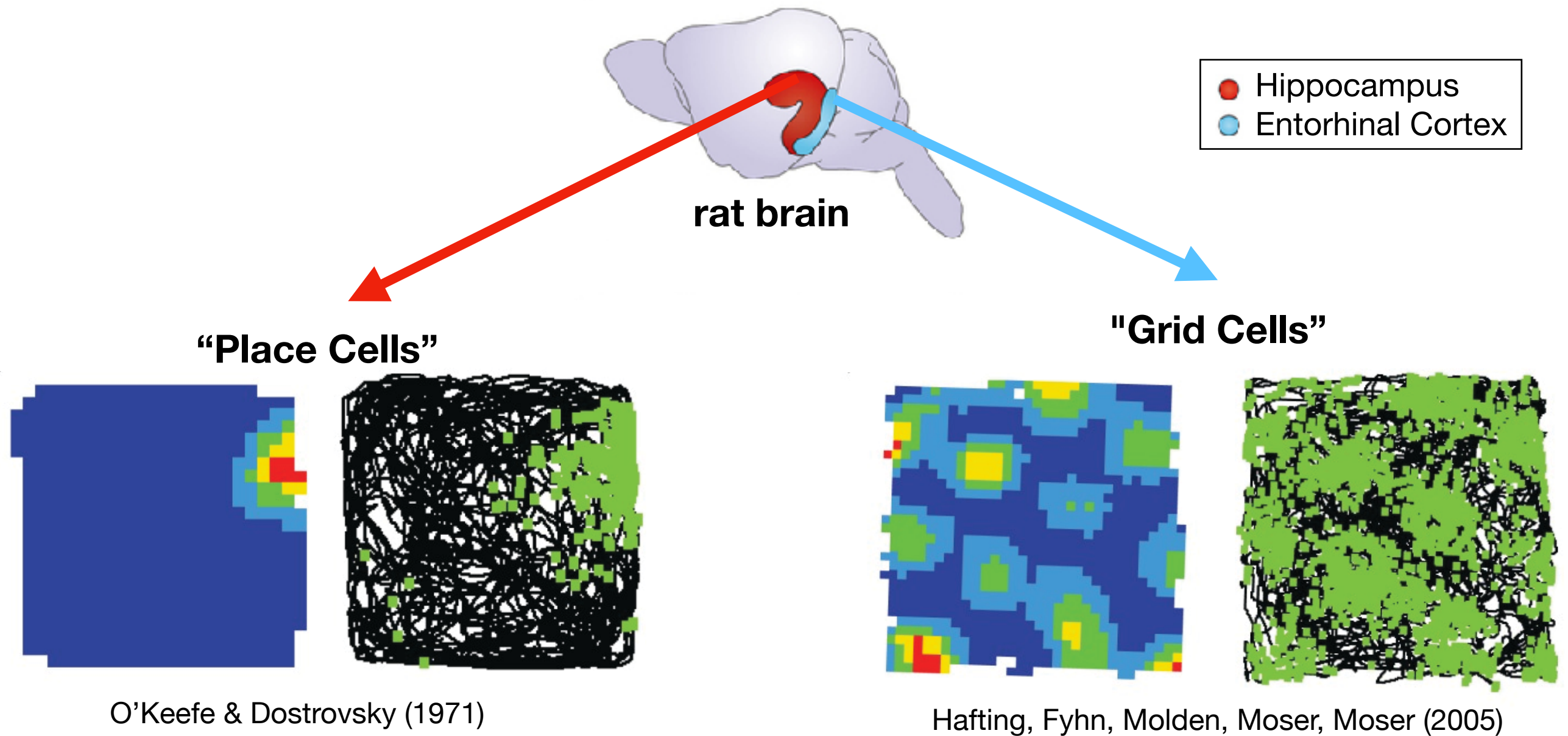
**“We assert that the central office itself is far more like a map control room than it is like an old fashioned telephone exchange.**

The stimuli, which are allowed in, are not connected by just simple one-to-one switches to the outgoing response. Rather, the incoming impulses are usually worked over and elaborated in the central control room into a tentative, cognitive-like map of the environment. And it is this tentative map, indicating routes and paths and environmental relationships, which finally determines what responses, if any, the animal will finally release”  
(Tolman 1948, p.192).

# Hippocampus as a “Cognitive Map”

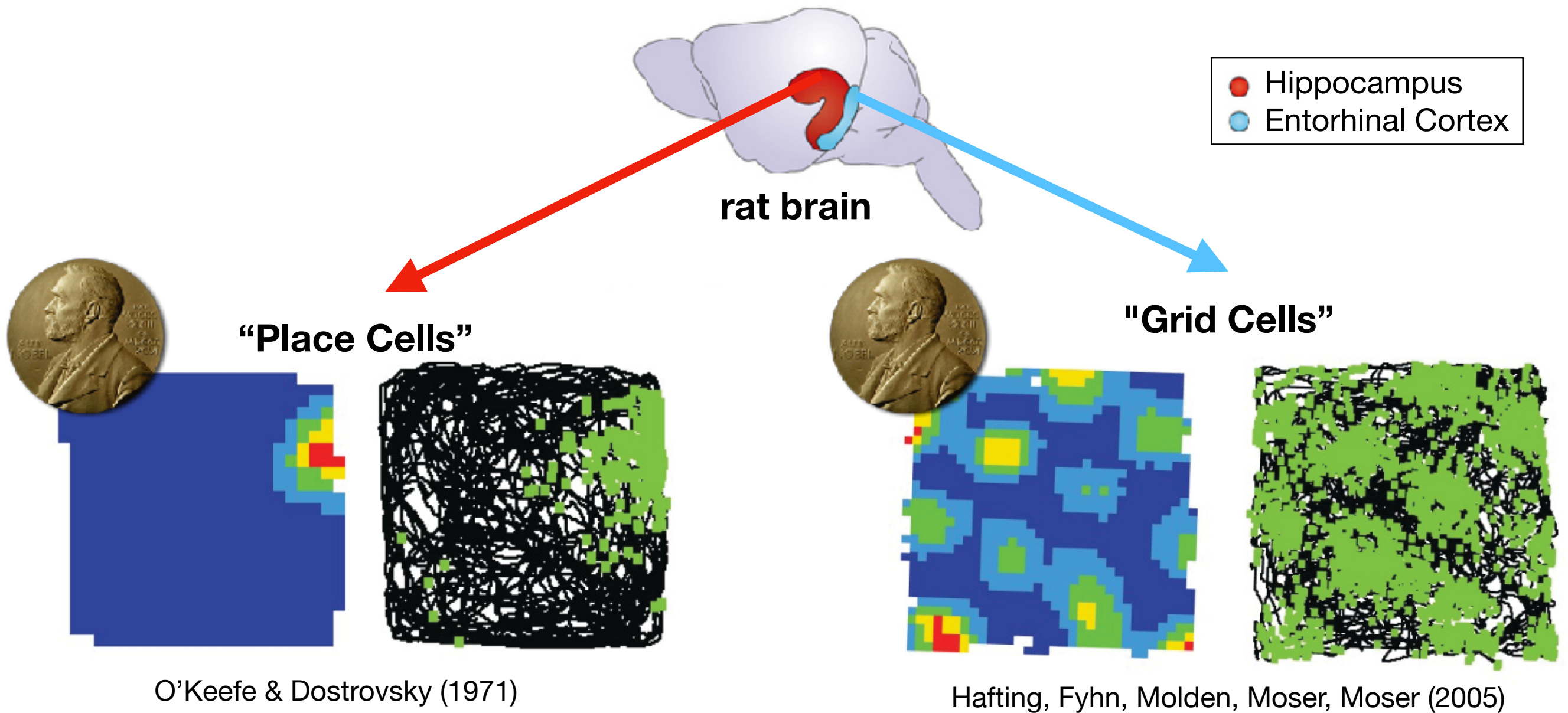


# Hippocampus as a “Cognitive Map”

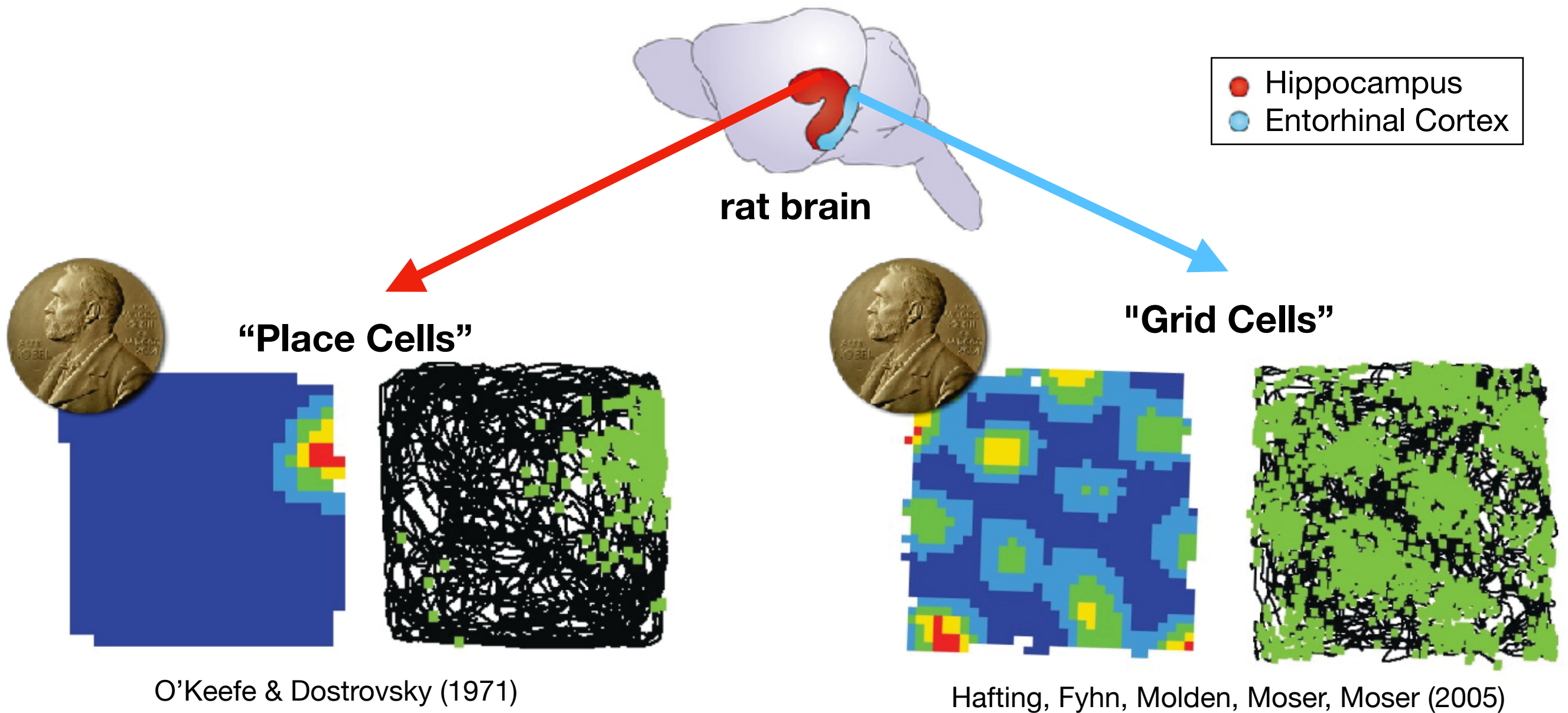




# Hippocampus as a “Cognitive Map”



# Hippocampus as a “Cognitive Map”



*See also: cells encoding head direction, boundaries, objects....*

# "Place Cells"

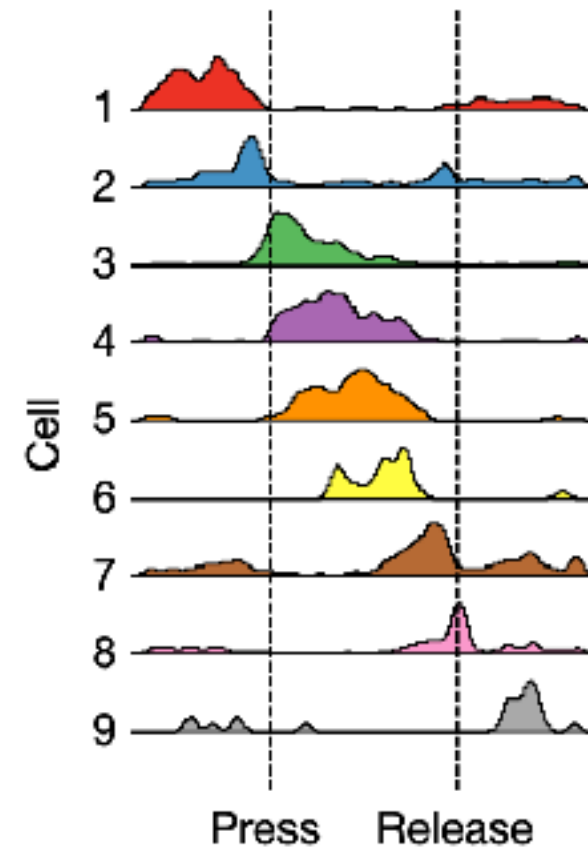
# "Place Cells"

- Classically selective for physical location

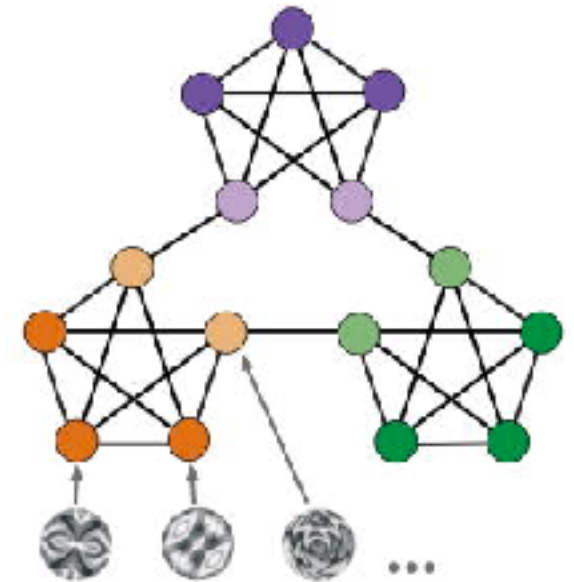
# "Place Cells"

- Classically selective for physical location
- Encode non-spatial variables too

**e.g. Sound**  
Aronov et al (2017)



**Graph Clusters**  
Schapiro et al (2016)

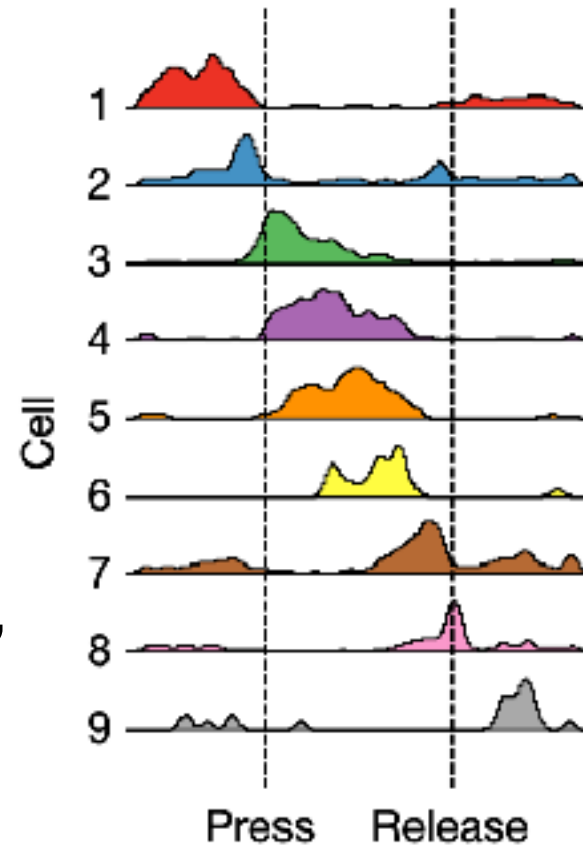




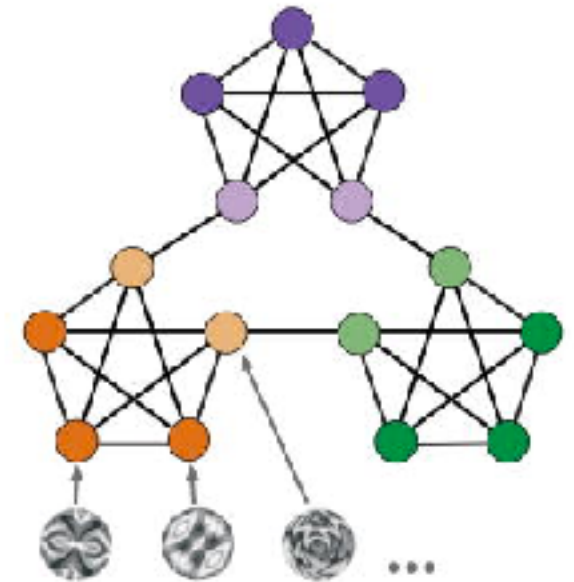
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- Classically selective for physical location
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- Place fields are altered by experience, in particular CA1 place fields long-lasting "prospective" shifts

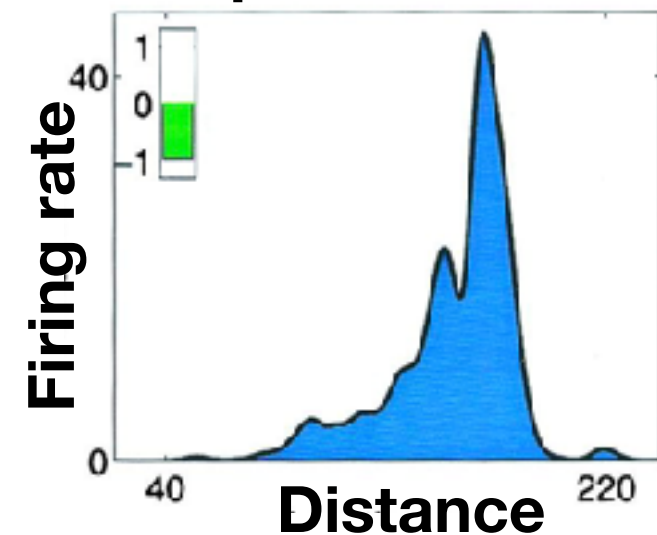
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**Prospective Shifts**

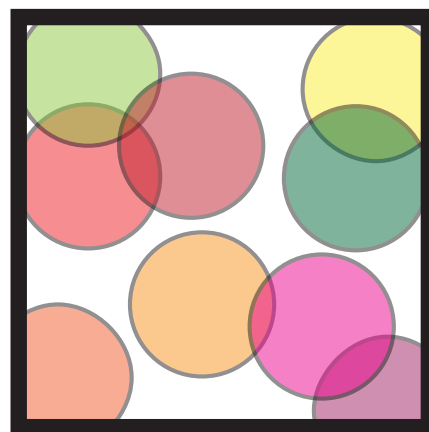


Mehta et al 2000

# "Place Cells"

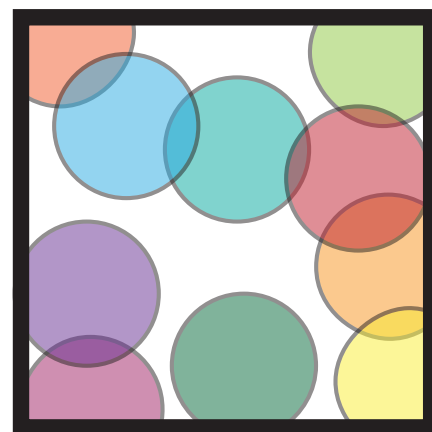
- Classically selective for physical location
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- "Global remapping"

Global remapping  
of hippocampal place cells



**room 1**

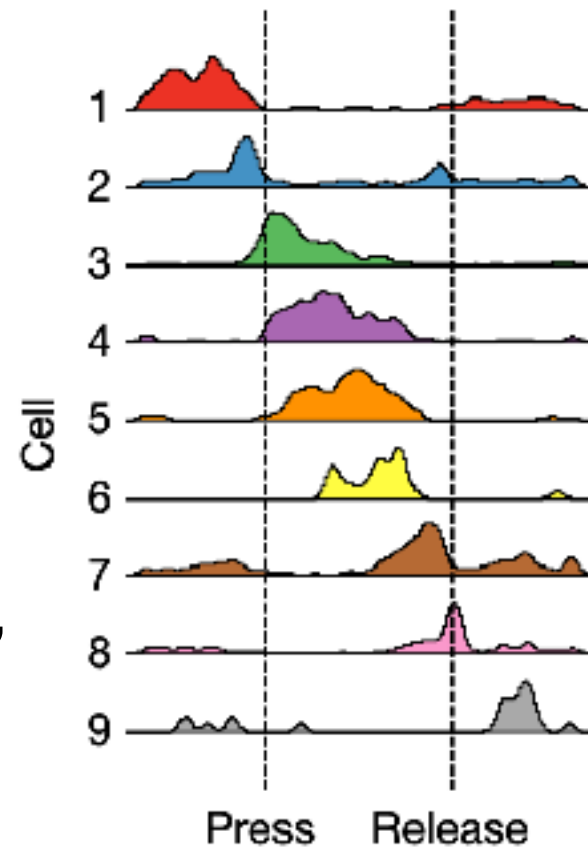
Behrens *et al* 2019



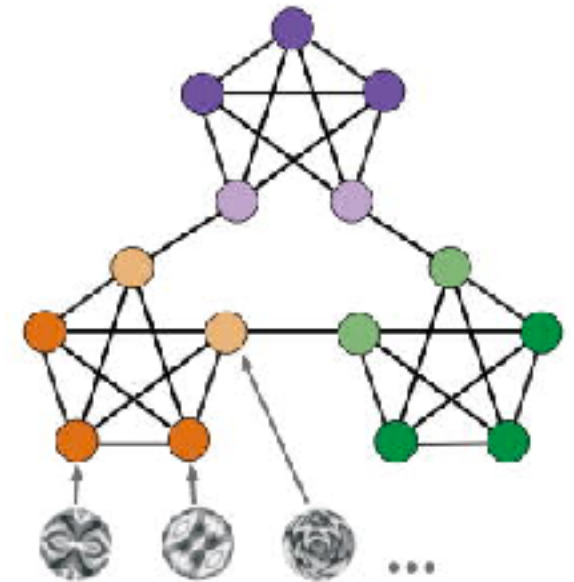
**room 2**

...  
Fields for different  
place cells

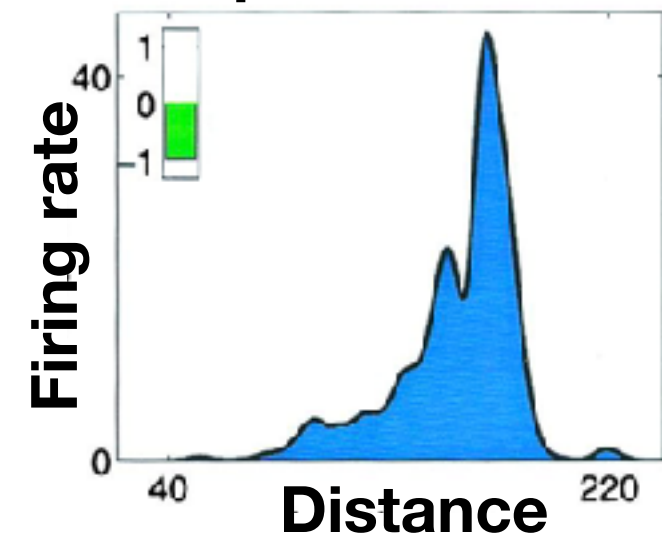
e.g. Sound  
Aronov *et al* (2017)



Graph Clusters  
Schapiro *et al* (2016)



Prospective Shifts



Mehta *et al* 2000

# Grid cells

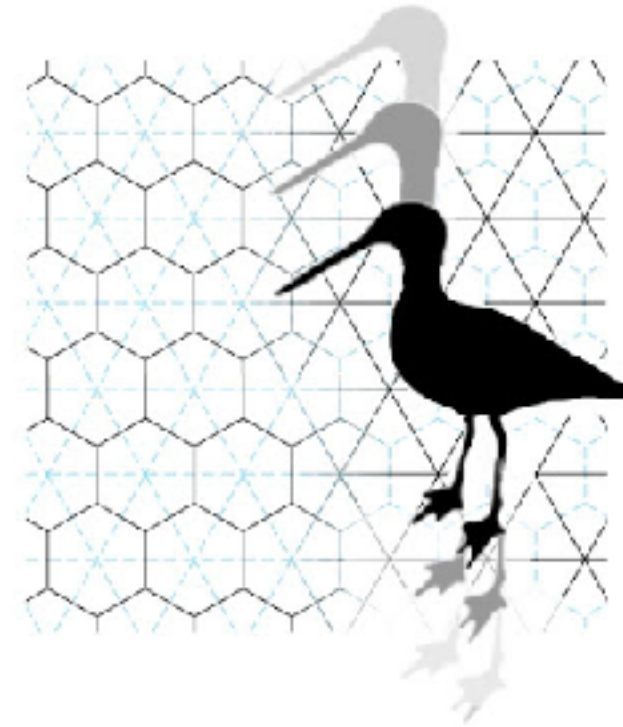
# Grid cells

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- Also encode non-spatial variables

e.g. “Stretchy birds”  
Continescu et al (2017)

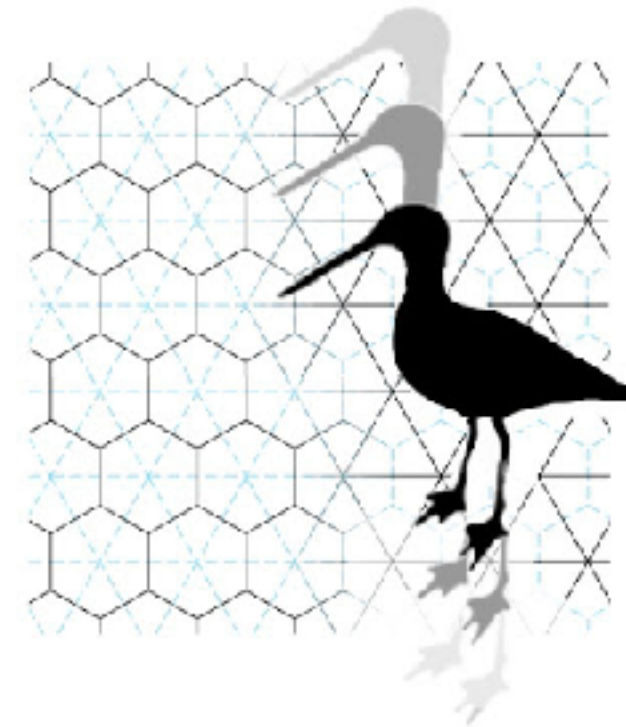




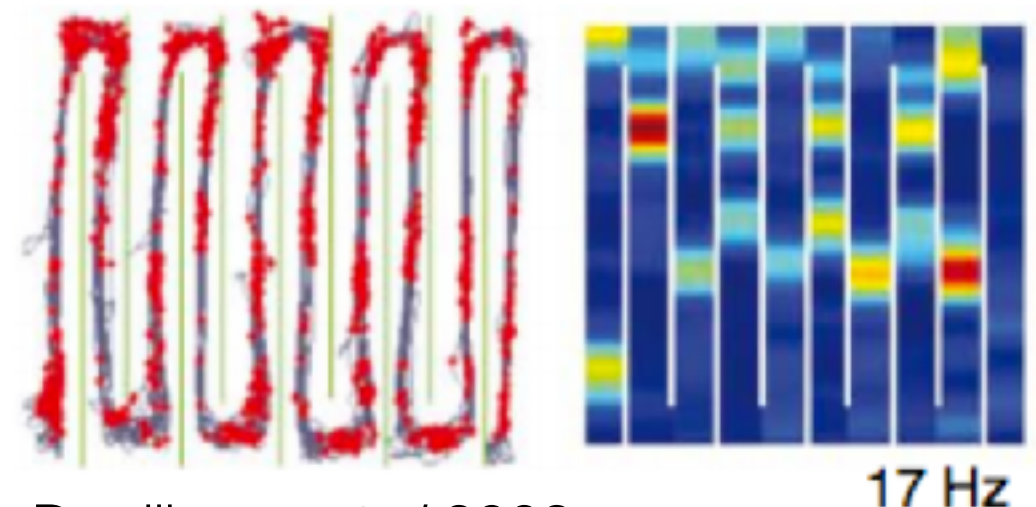
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- Firing patterns are spatially periodic
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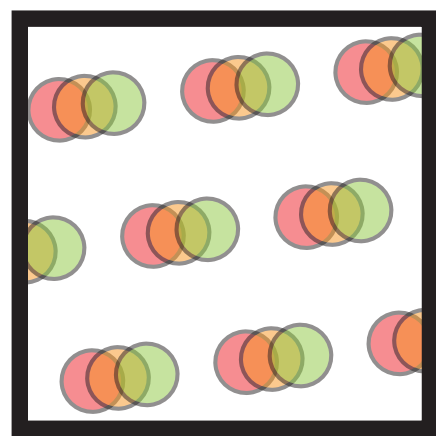
## Receptive Field in Hairpin Maze



Derdikman et al 2009

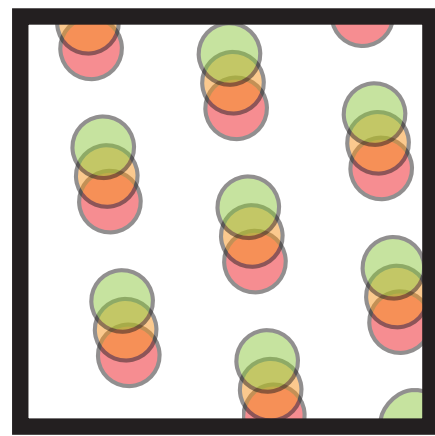
# Grid cells

- Firing patterns are spatially periodic
- Also encode non-spatial variables
- Firing properties influenced by boundaries of environment
- No global remapping



**room 1**

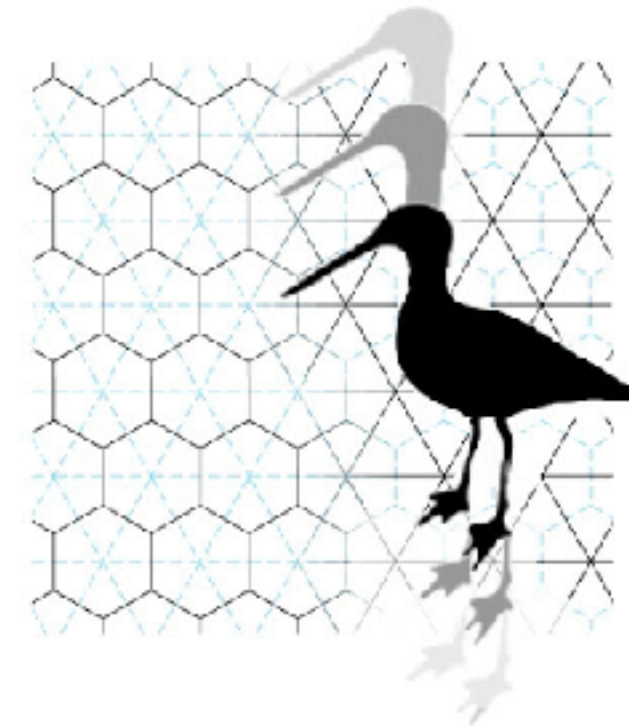
Behrens *et al* 2019



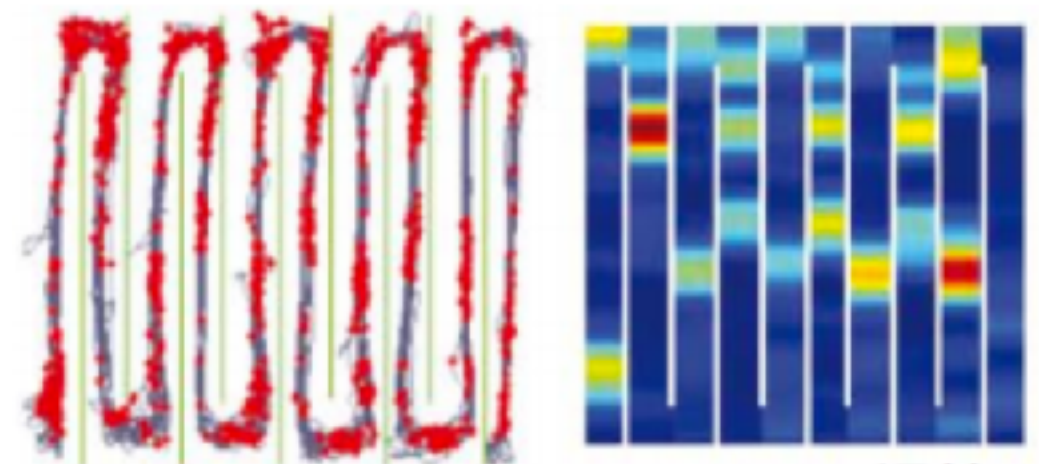
**room 2**

Fields for different  
grid cells

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Derdikman *et al* 2009

17 Hz

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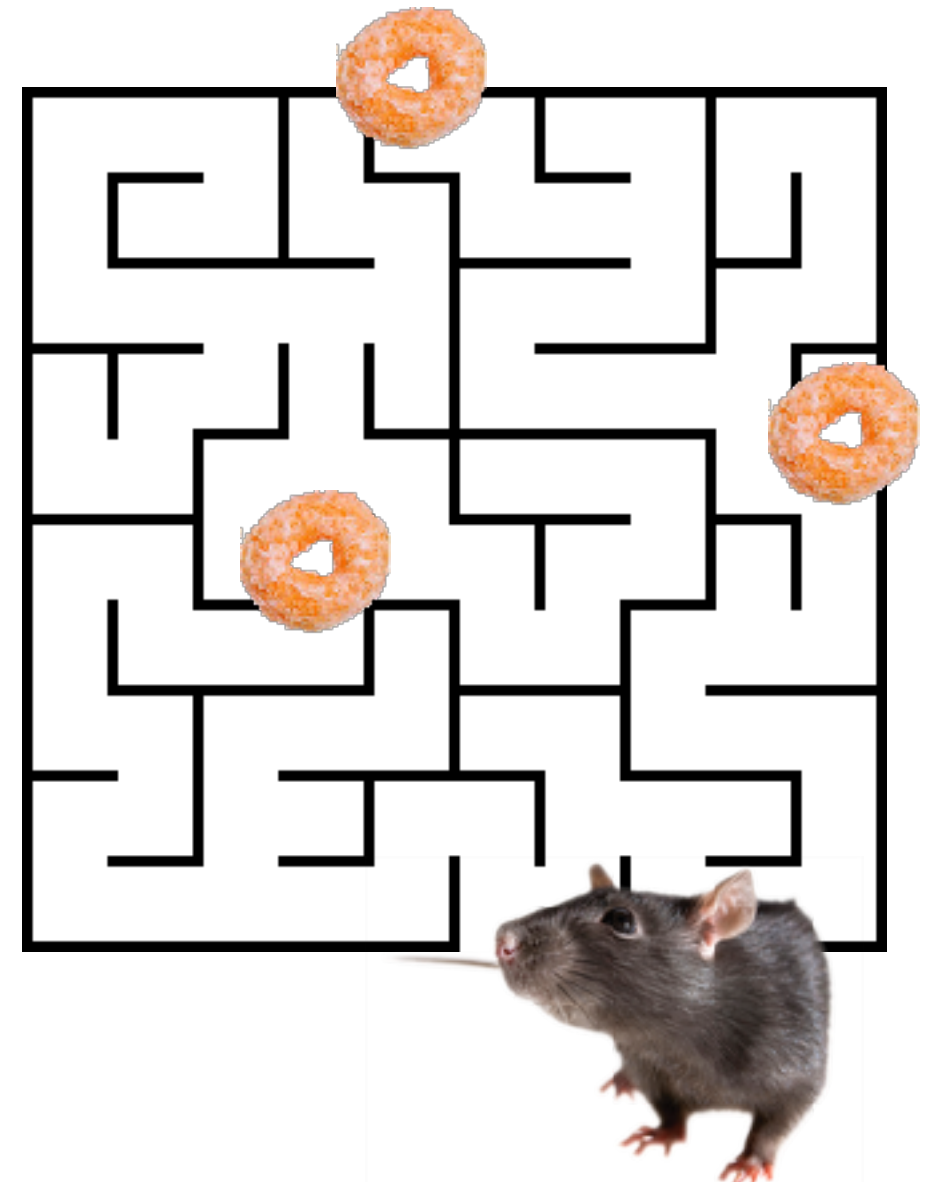
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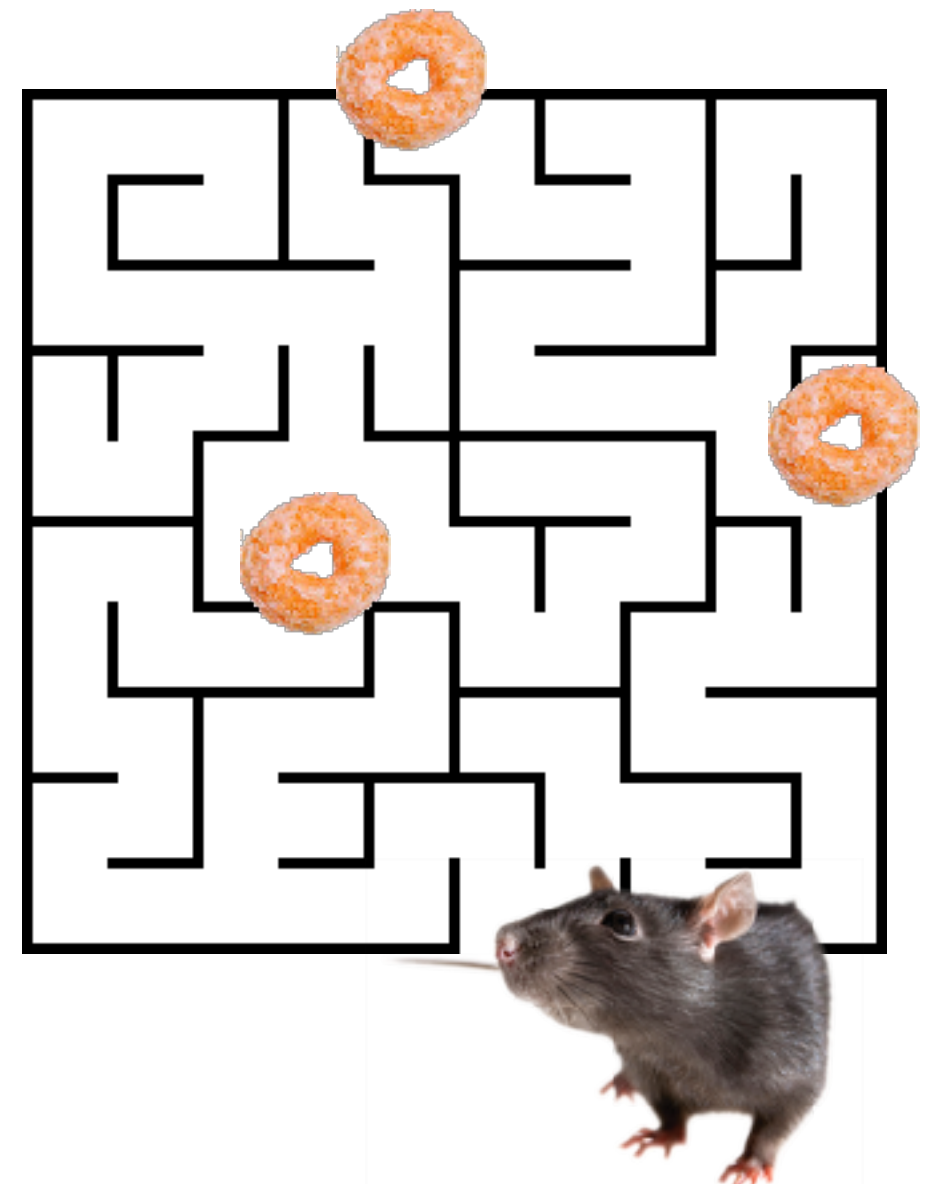
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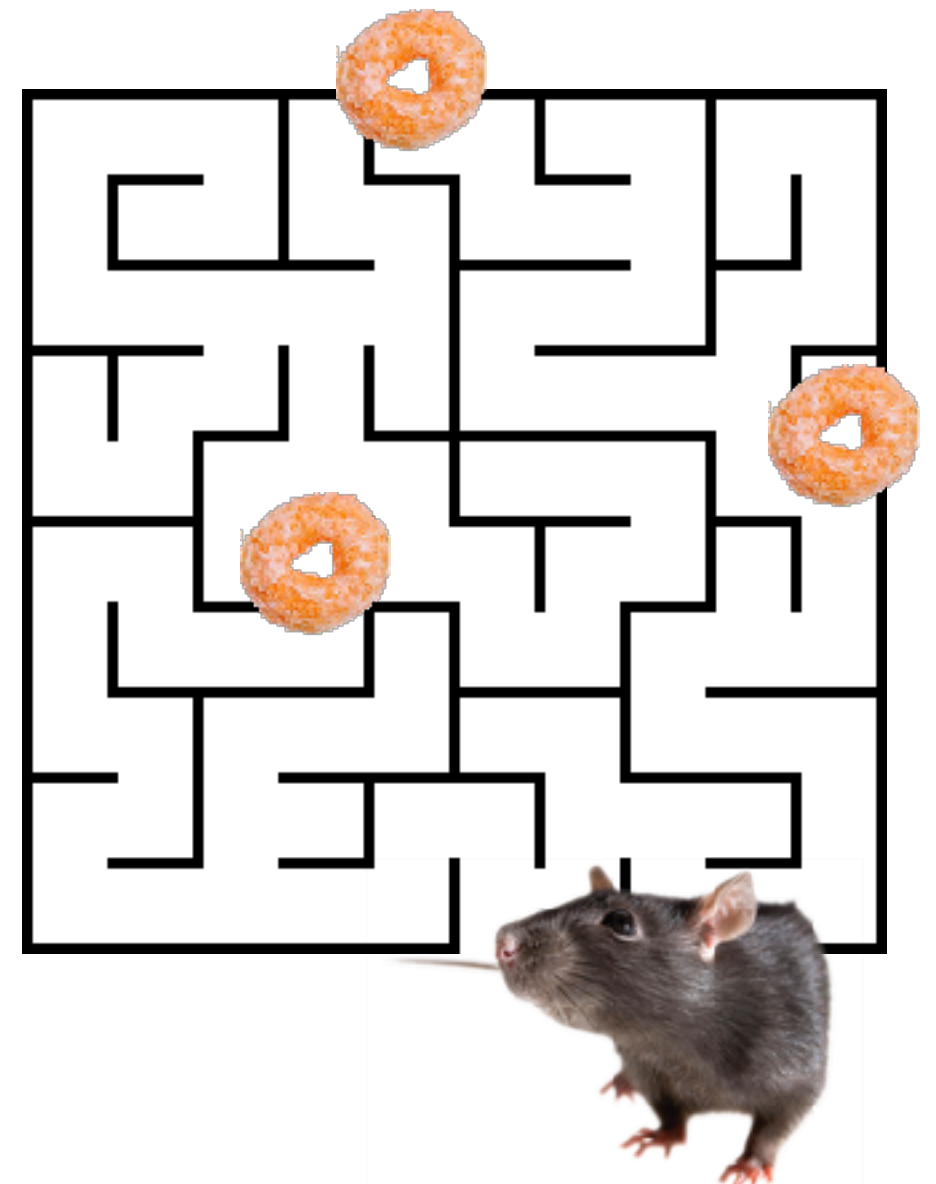
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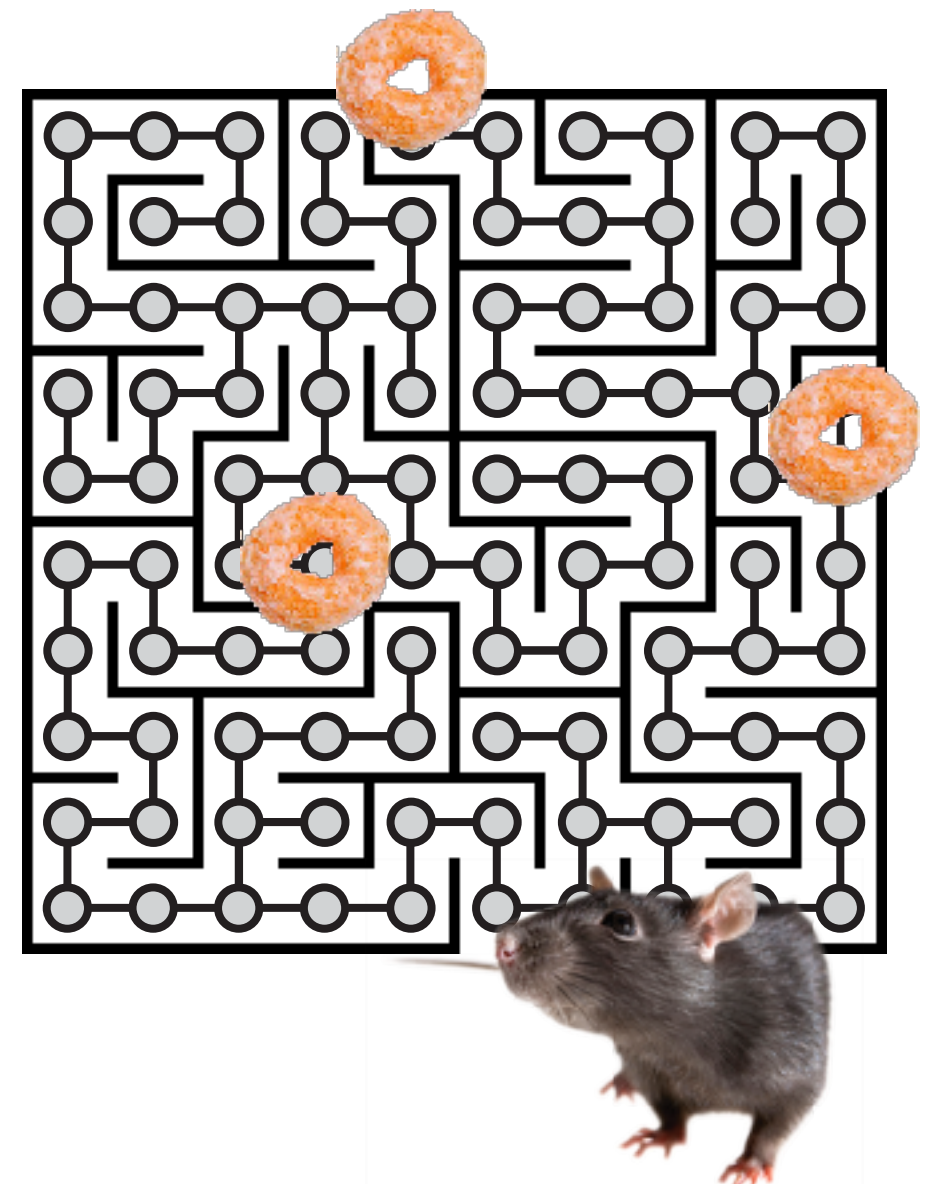
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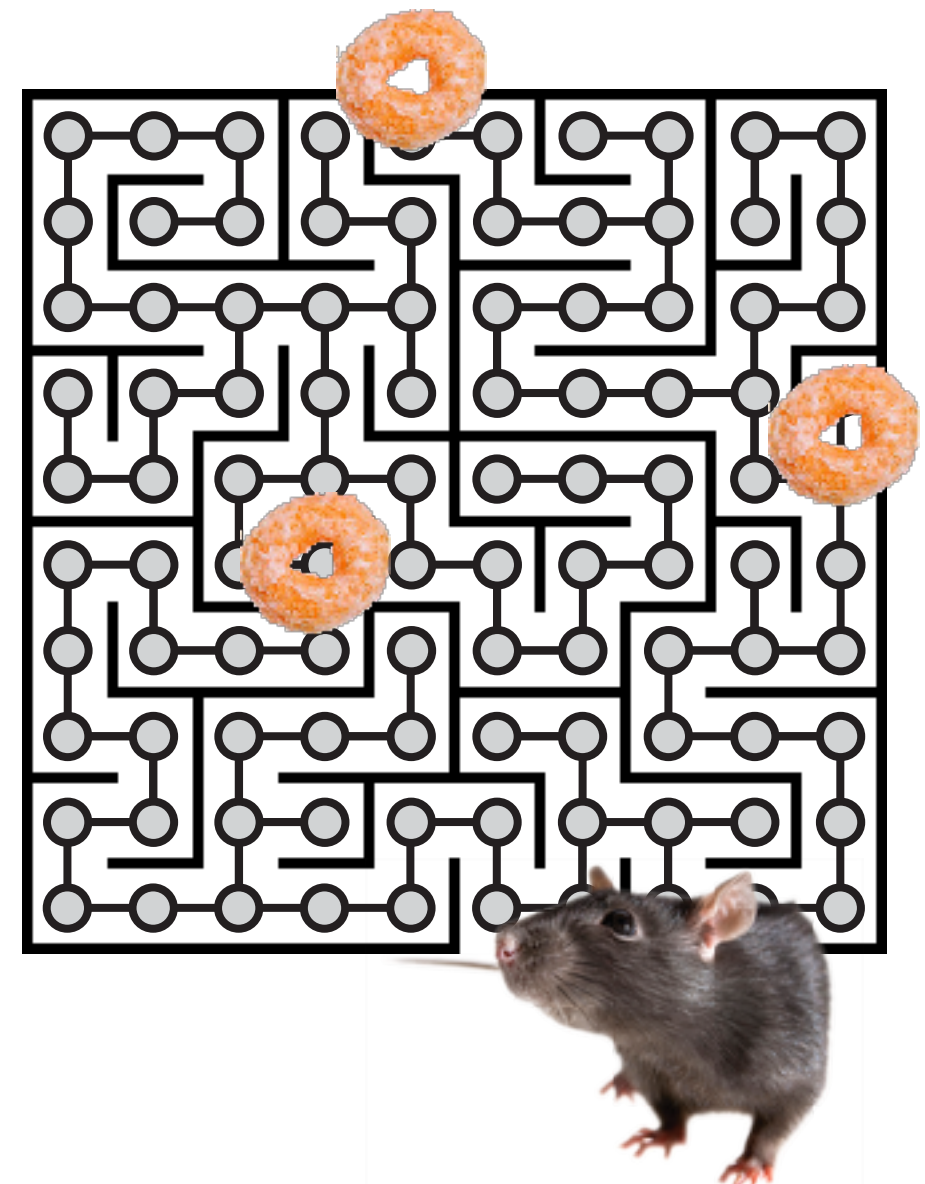
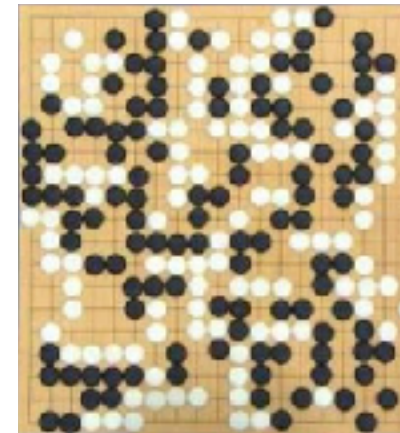
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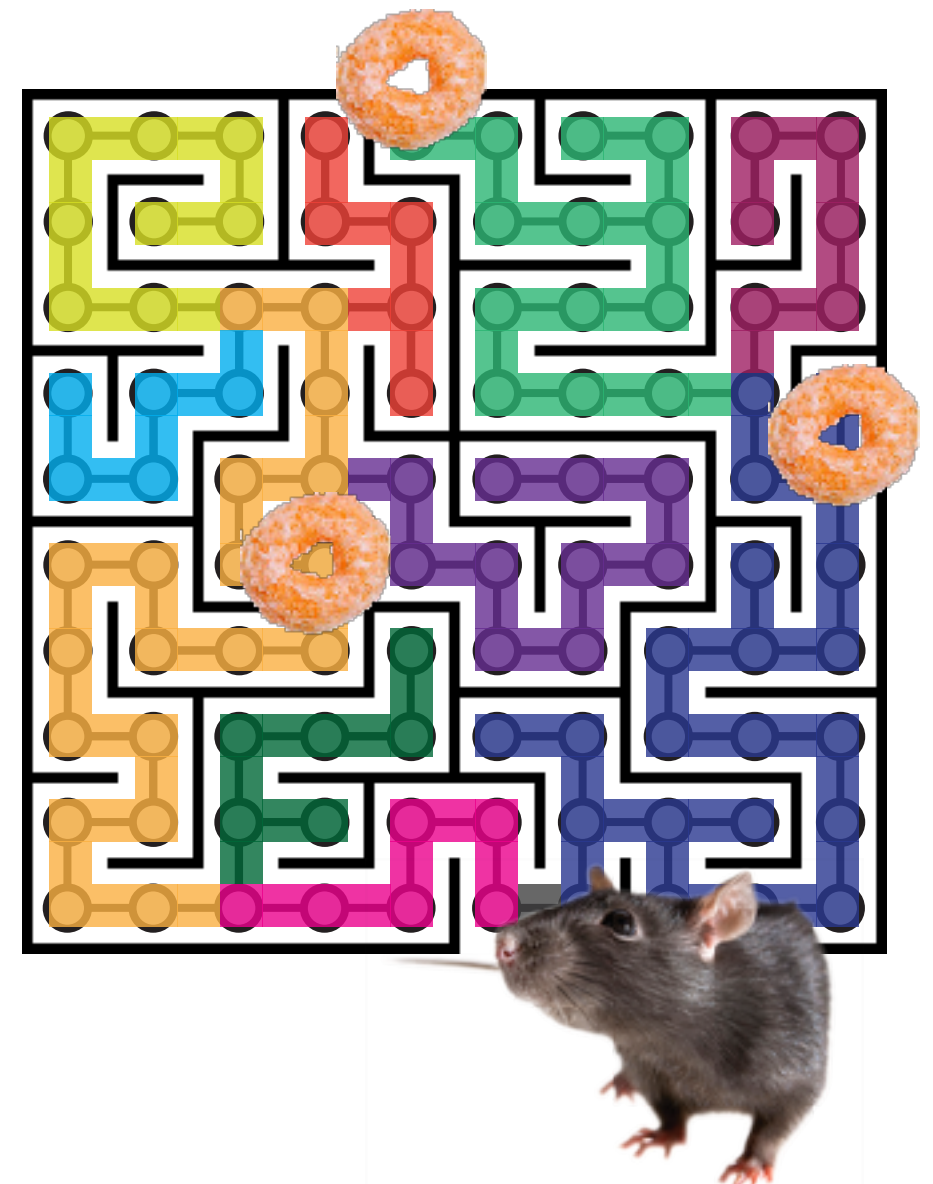
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- How do neural representations capture structure and use it for efficient + flexible learning and inference?
- How do we get machines to do that?



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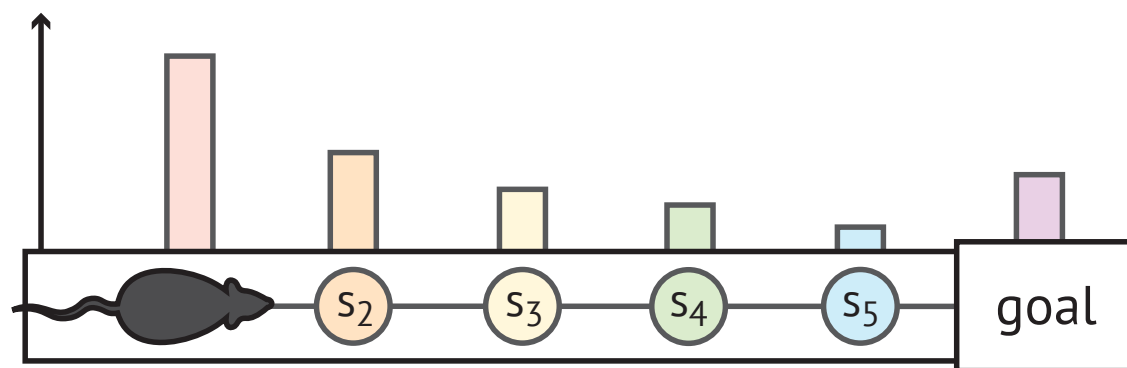
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## Predictions

Represent states in terms of the predictions they make



## Predictive Representations:

Predictive State Representations (Singh et al 2012)  
Successor features (Dayan 1993, Barreto et al 2016, Kulkarni et al 2016)  
Universal Value Function Approximators (Schaul et al 2015)  
Contrastive predictive coding (Oord et al 2018)  
MERLIN (Wayne et al 2018)

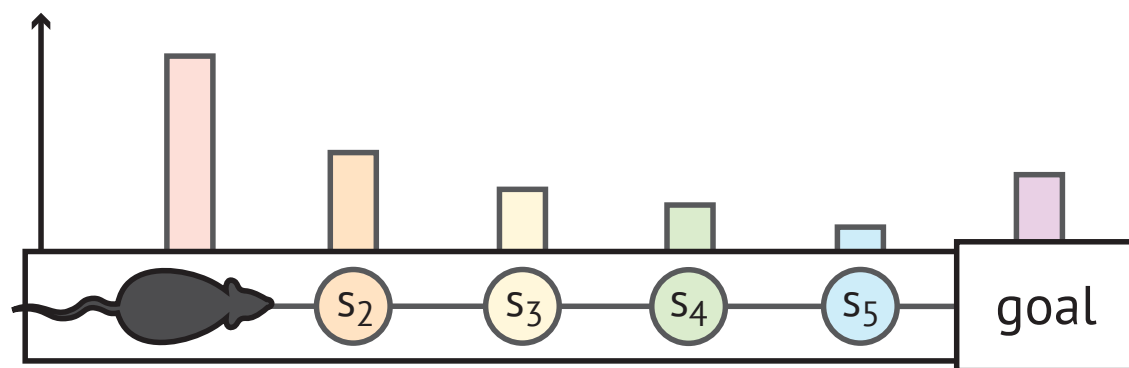
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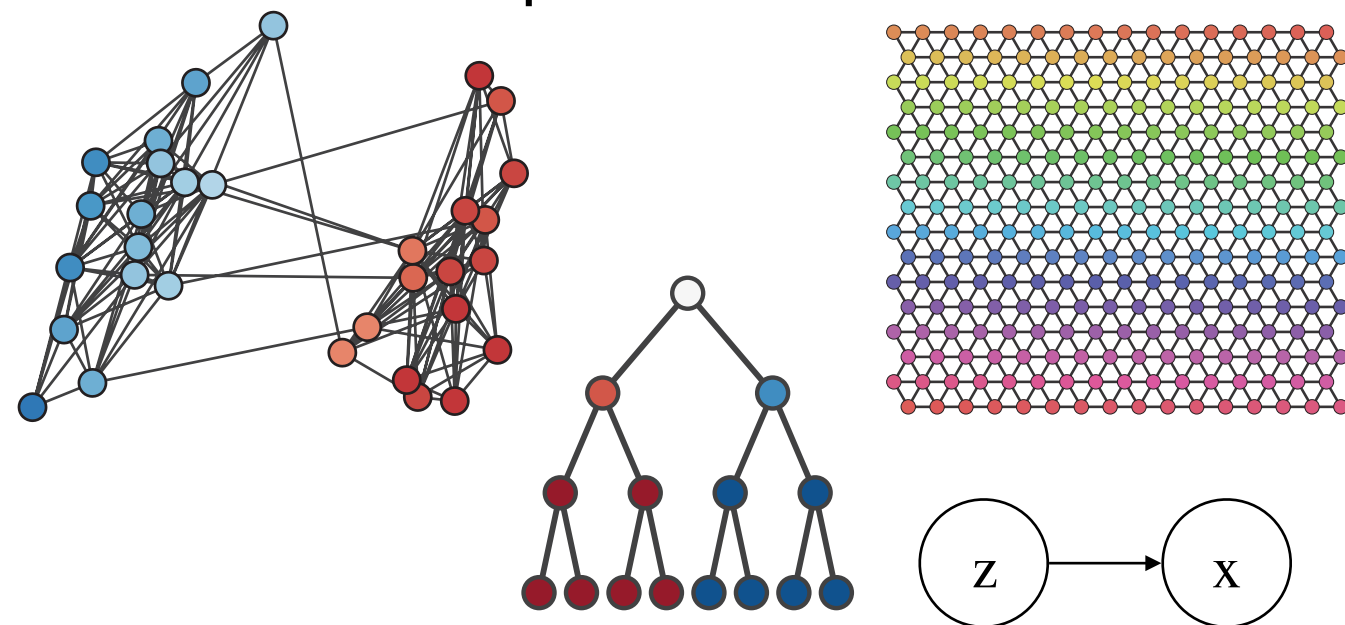


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Learn as short description as possible



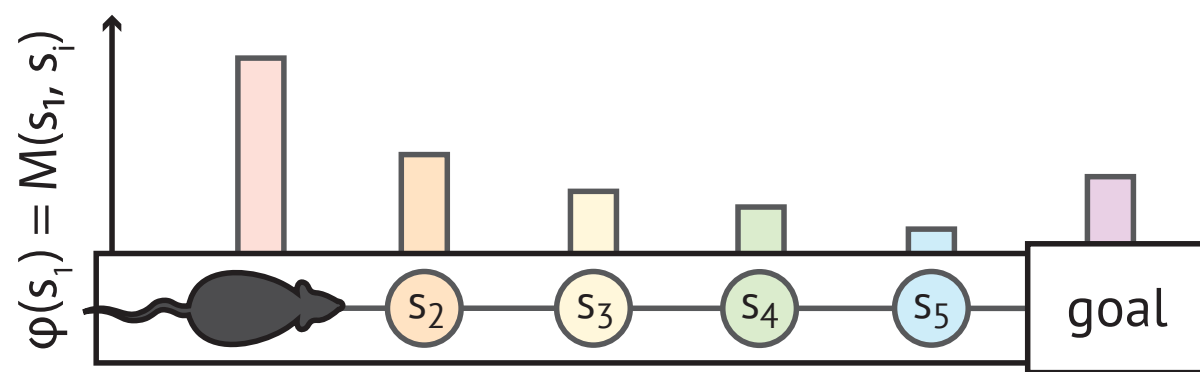
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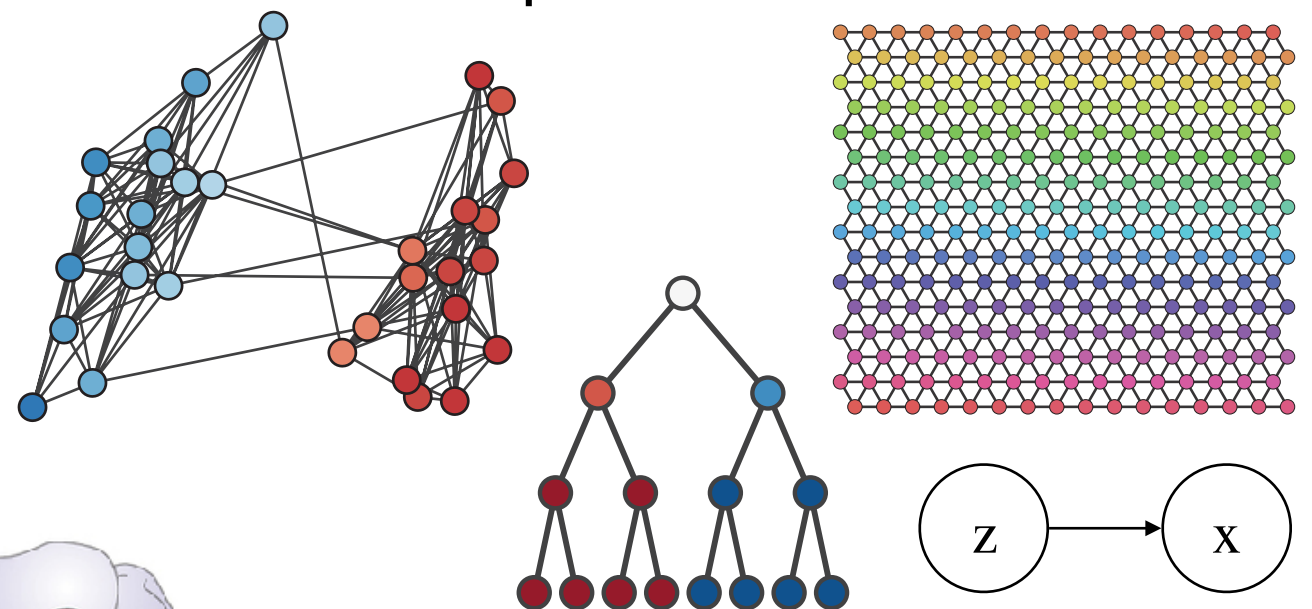
## Predictions

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## Compression

Learn as short description as possible



## Place Cells

For each state, learn how often you will visit other states?



## Grid cells

Capture low-dimensional structure among predictions

# Predictive representations



# Predictive representations

$$V(s) = \mathbf{E} [ R(s_{t0}) + \gamma R(s_{t1}) + \gamma^2 R(s_{t2}) + \dots ]$$

value

add up expected rewards for states over time  
discount with  $\gamma < 1$  to prioritize immediacy

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$$V = \left( \sum_{t=0}^{\infty} \gamma^t \mathbf{T}^t \right) \mathbf{R}$$

expected visits  
different states

reward

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expected visits      reward  
different states

$$\mathbf{V} = \mathbf{M} \mathbf{R}$$

successor      reward  
representation  
matrix

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*Instantaneous  
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$$V = \left( \sum_{t=0}^{\infty} \gamma^t T^t \right) R$$

expected visits      reward  
different states

*Caches long-term  
effect of dynamics*

$$V = M R$$

successor      reward  
representation  
matrix

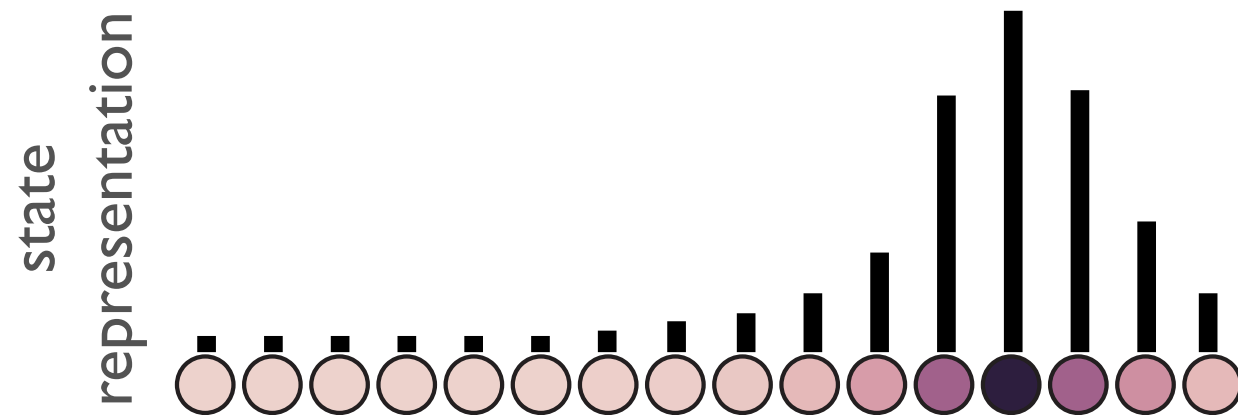
*Instantaneous  
reward*

# Successor representations in hippocampus



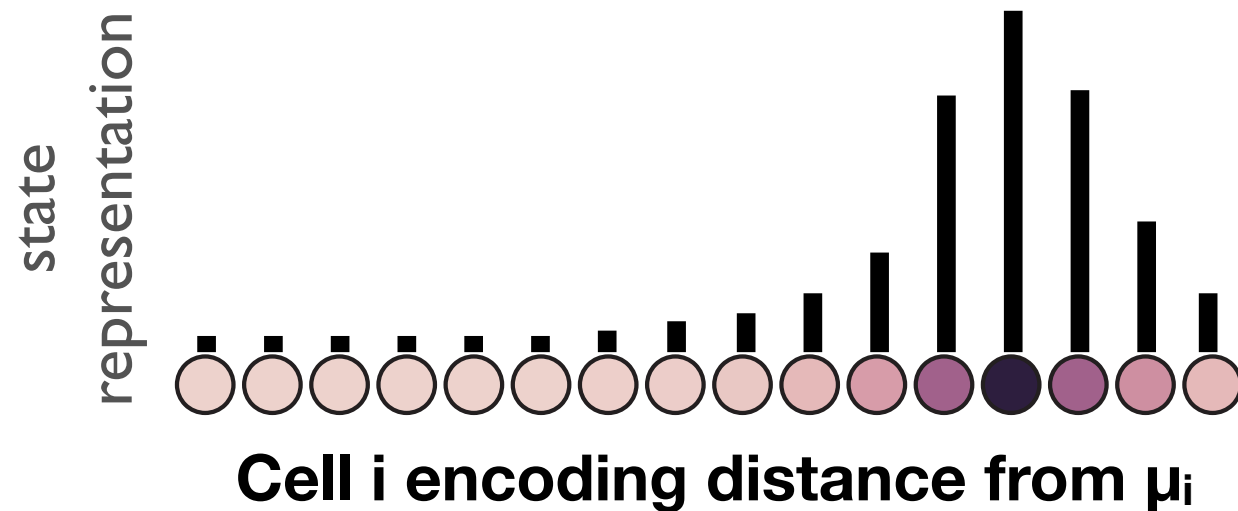
# Successor representations in hippocampus

## Place Representation



# Successor representations in hippocampus

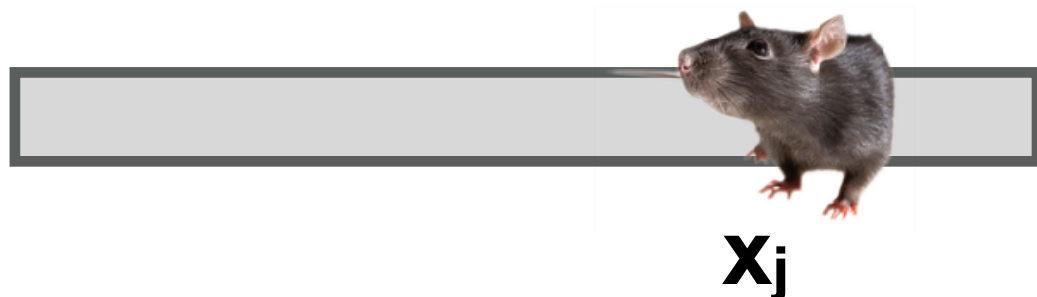
## Place Representation



Firing of cell  $i$   
At location  $j$

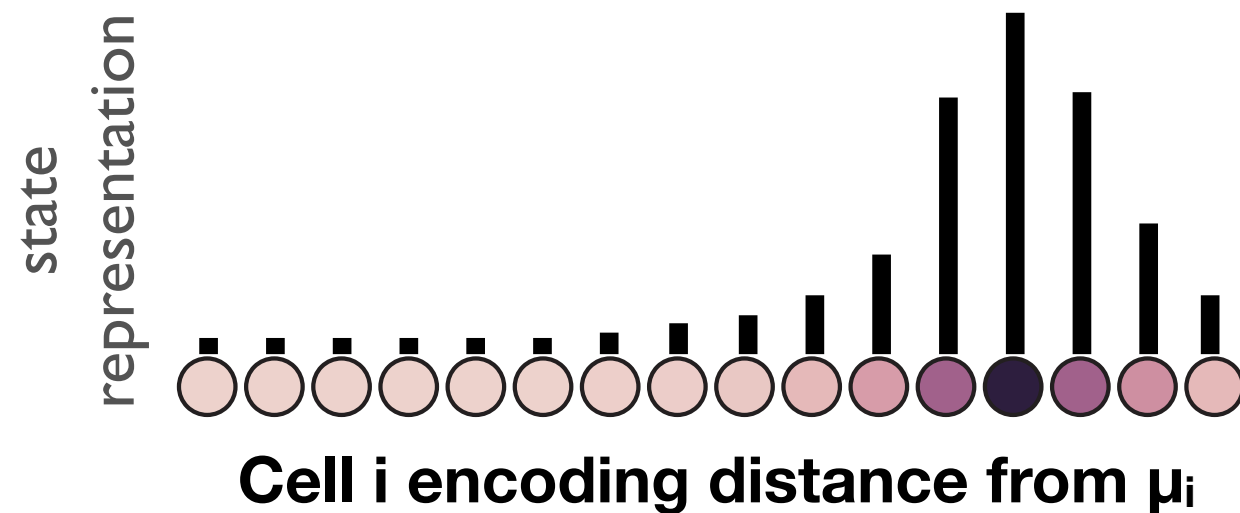
$$A e^{-\left(\frac{x_j - \mu_i}{2\sigma}\right)^2}$$

**Euclidean Gaussian with center  $i$**   
centered at  $\mu_i$ , width  $\sigma$ , height  $A$



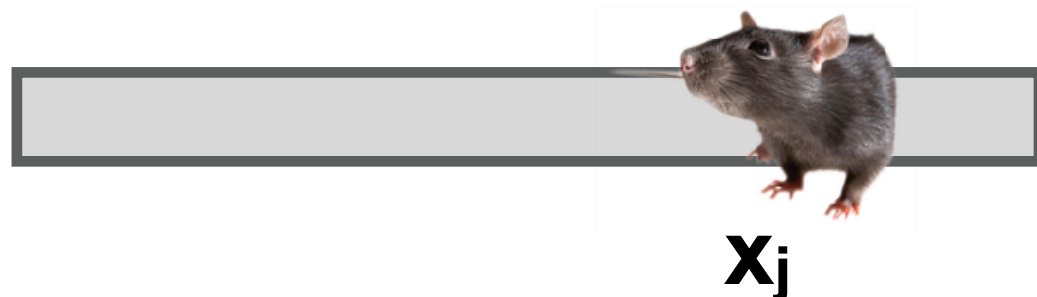
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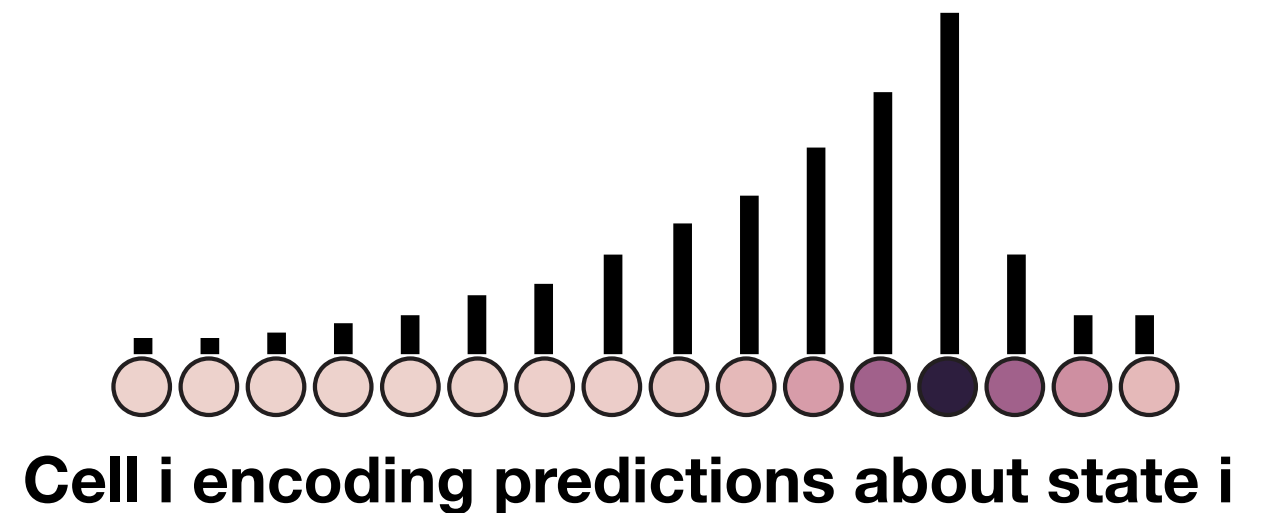


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**Euclidean Gaussian with center i**  
centered at  $\mu_i$ , width  $\sigma$ , height A

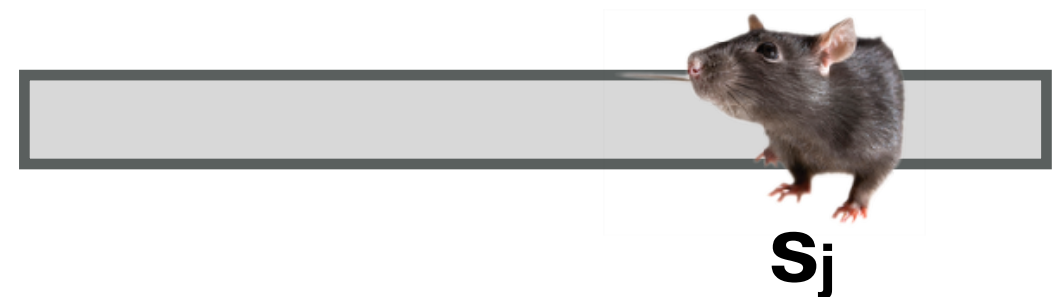


## Successor Representation



row j of matrix  $M = \sum_{t=0}^{\infty} \gamma^t T^t$

**Expected #visits to state i**  
Discount  $\gamma$

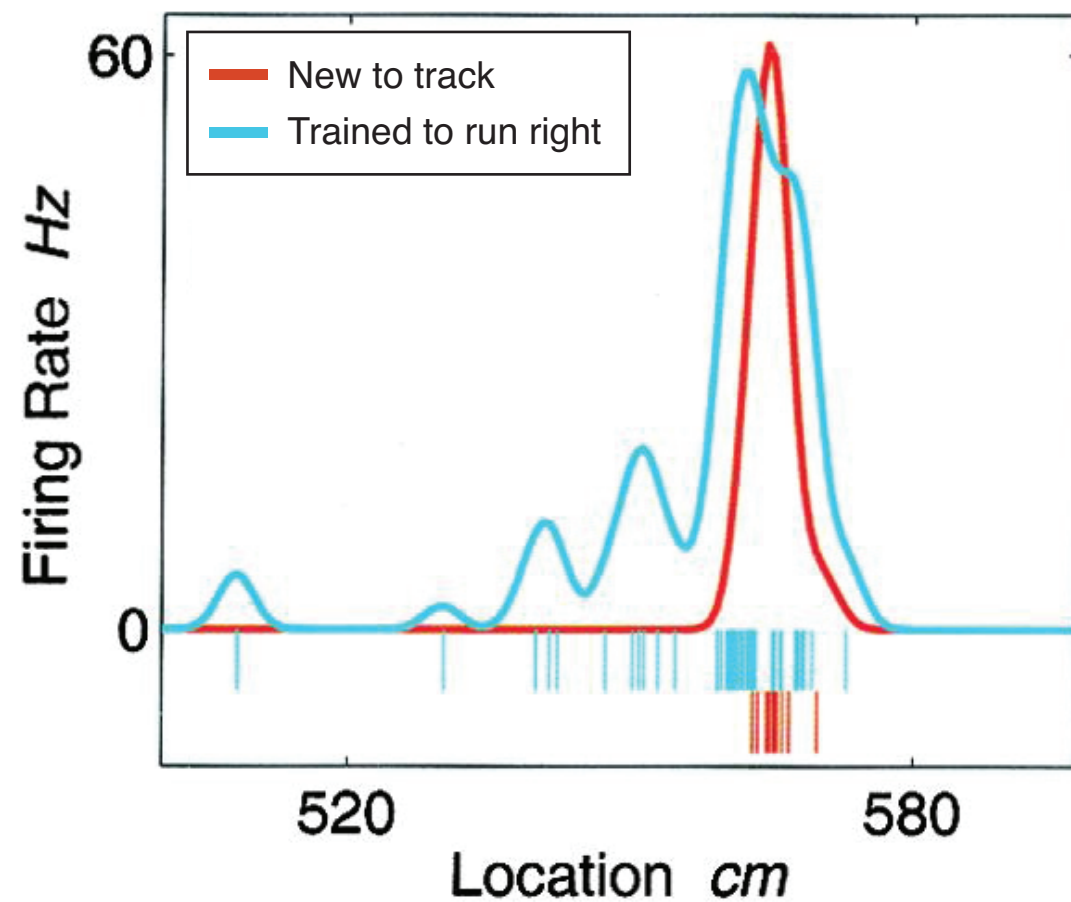


# Place fields along a track

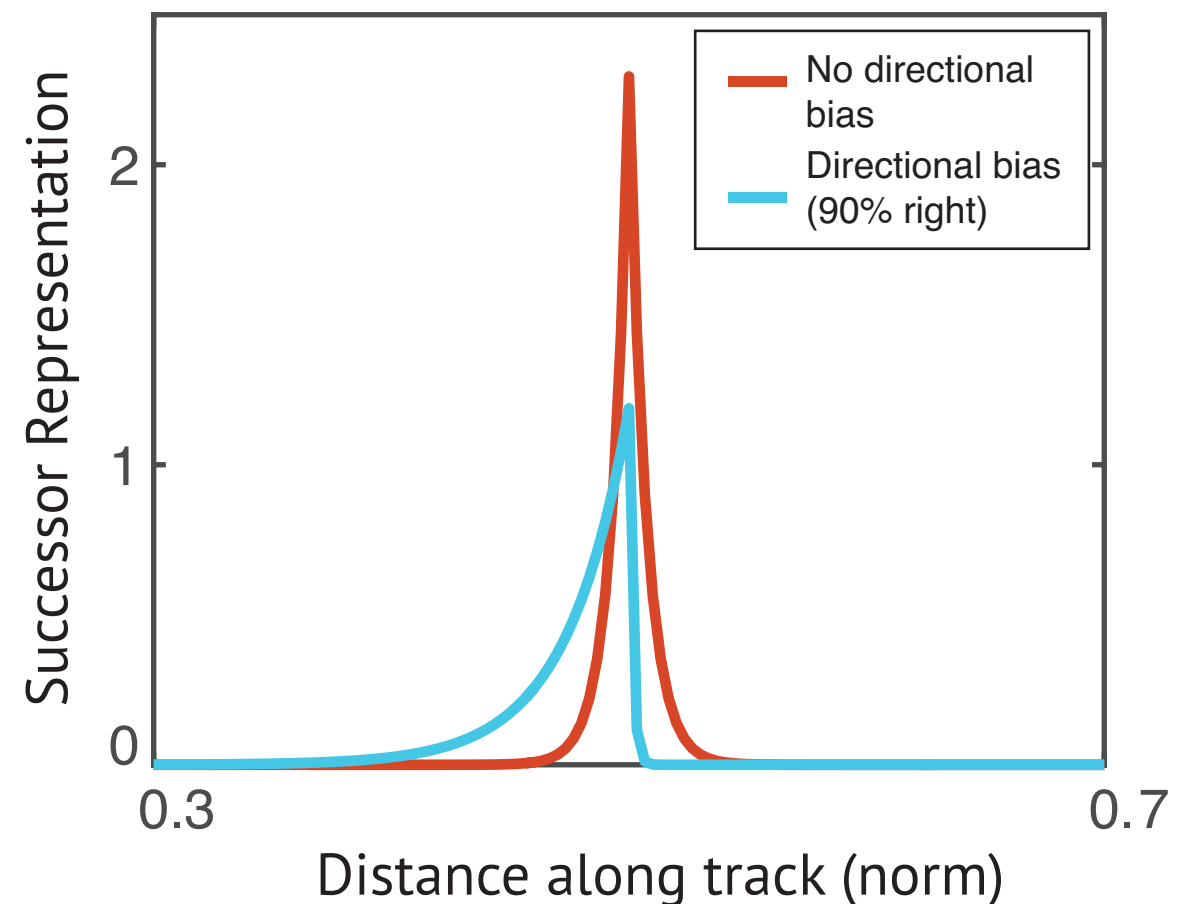
## Experiment

## Model

### A Mehta *et al.* (2000)



### B Simulated SR Place cells

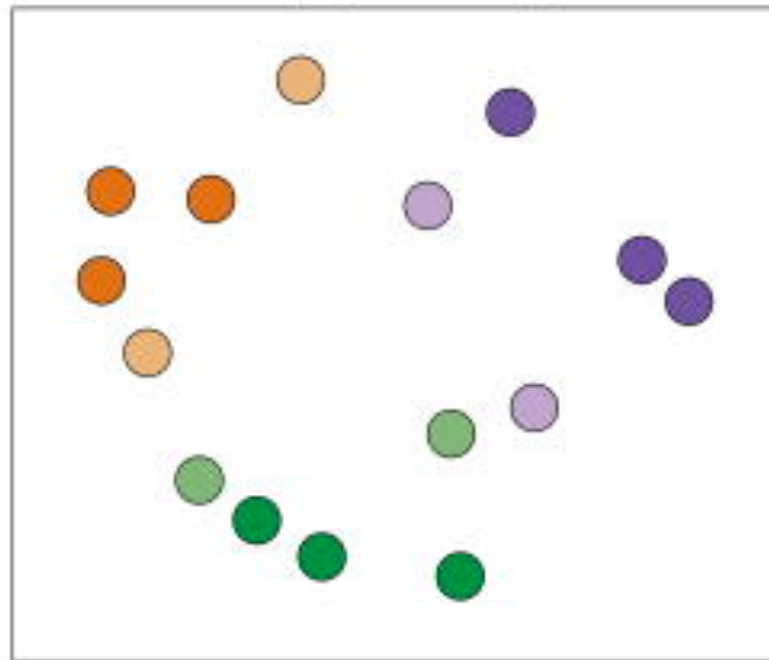
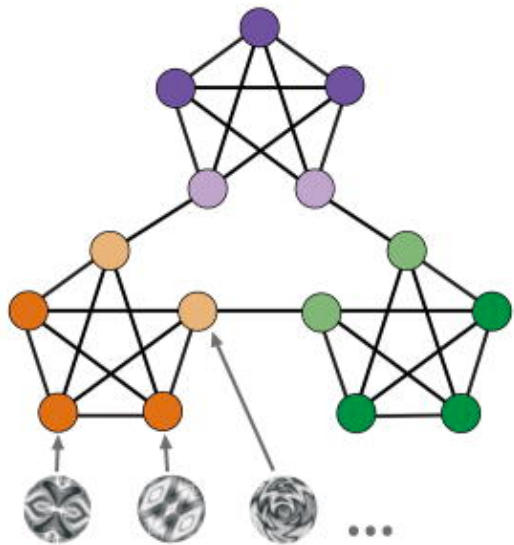


direction of motion

# Clustering by temporal structure

## Experiment

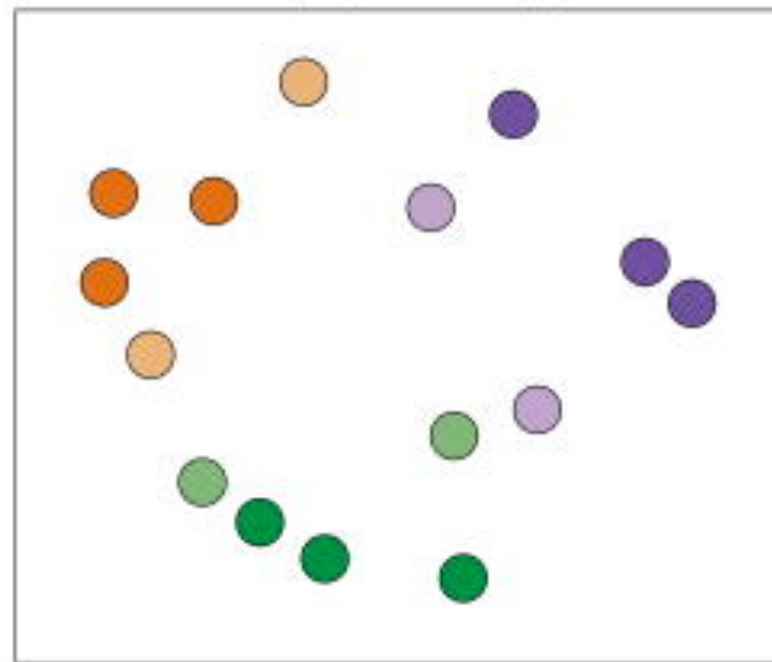
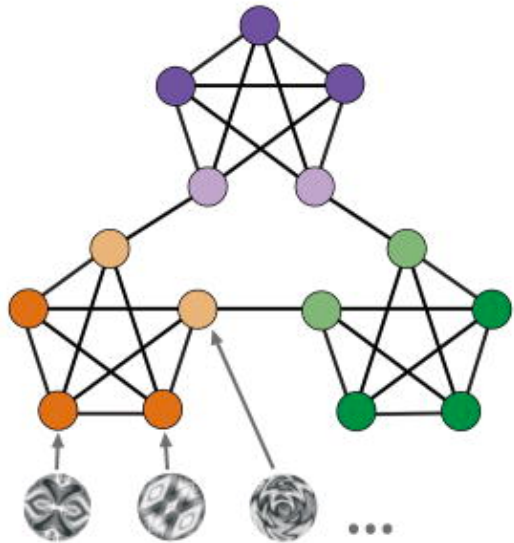
### Bilateral hippocampus MDS



# Clustering by temporal structure

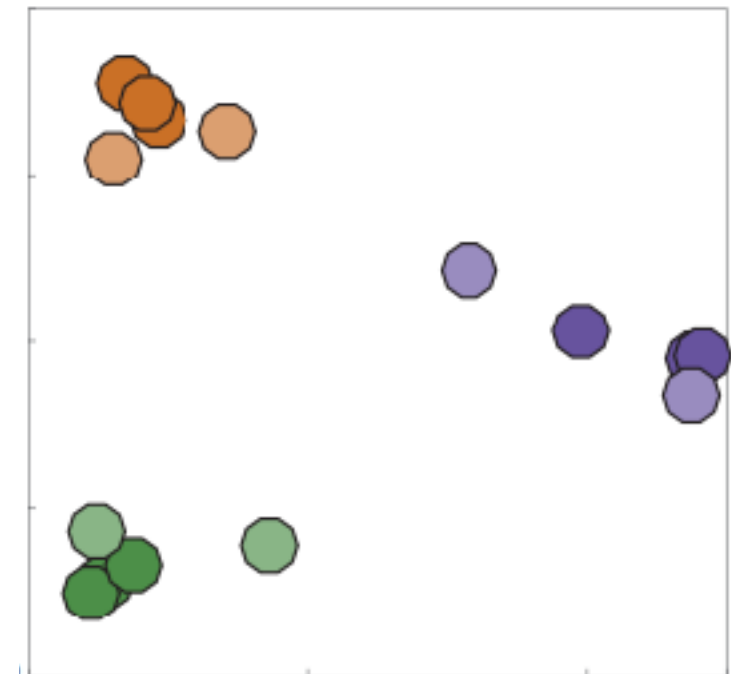
Experiment

Bilateral hippocampus  
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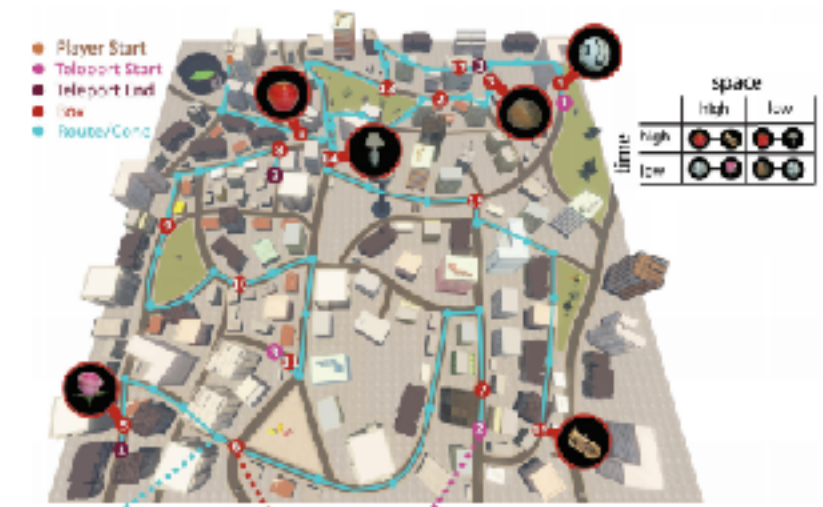
Simulation

Successor representation  
MDS

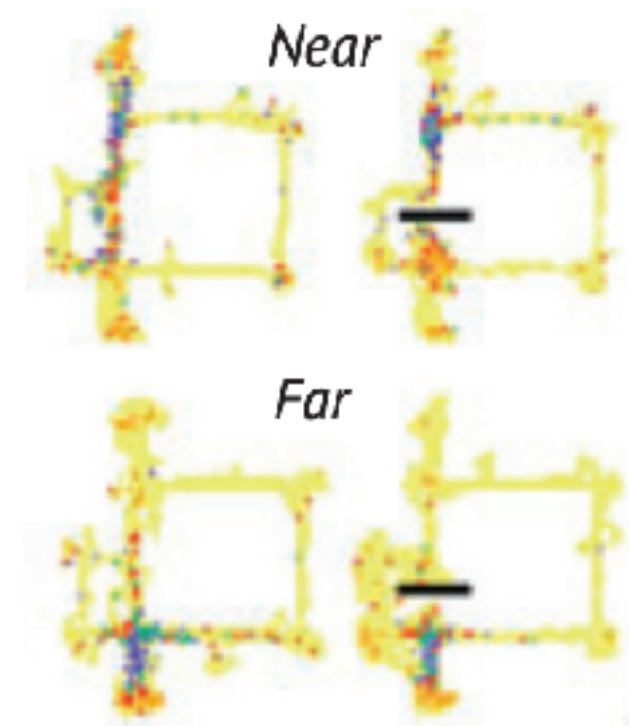


# And more!

- Mixed spatiotemporal distance effects (Decker et al 2016)
- Obstacle-induced effects (Alvernhe et al 2011)
- Some aspects of goal-related firing (Hollup et al 2001)
- For more successor representation experiments, see also: Russek et al (2017), Momennejad et al (2017), Garvert et al (2017)



CA1 Place fields





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- Flexible computation of value when reward changes\* because transition information is preserved in the successor representation
- \* **Caveat**: still not as flexible as model-based planning because predictions are averaged over different possible futures

$$\mathbf{V} = \mathbf{M} \mathbf{R}$$

*Caches long-term effect of dynamics* → successor representation matrix

← *Instantaneous reward*

reward

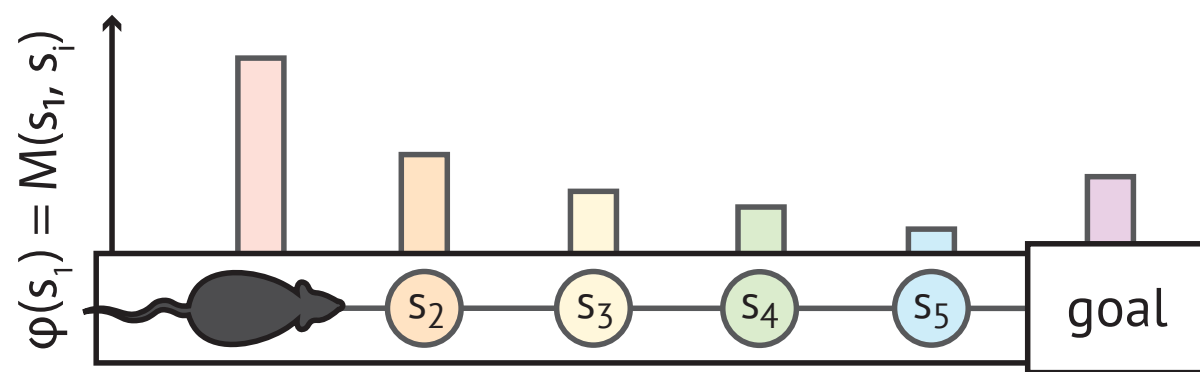
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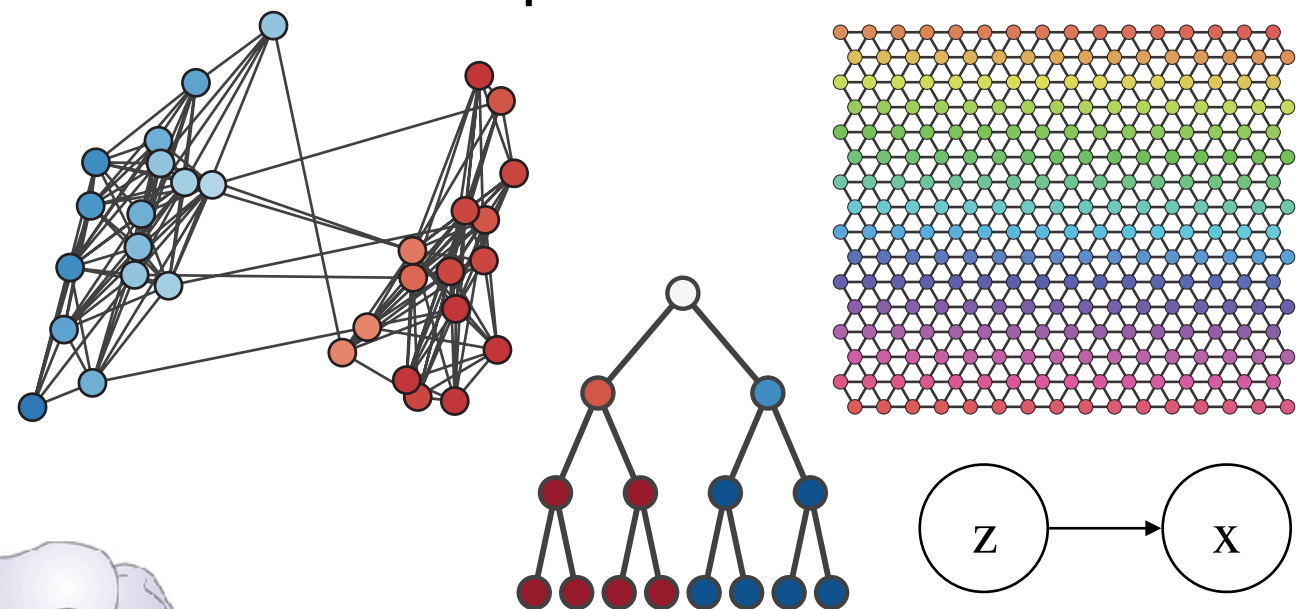
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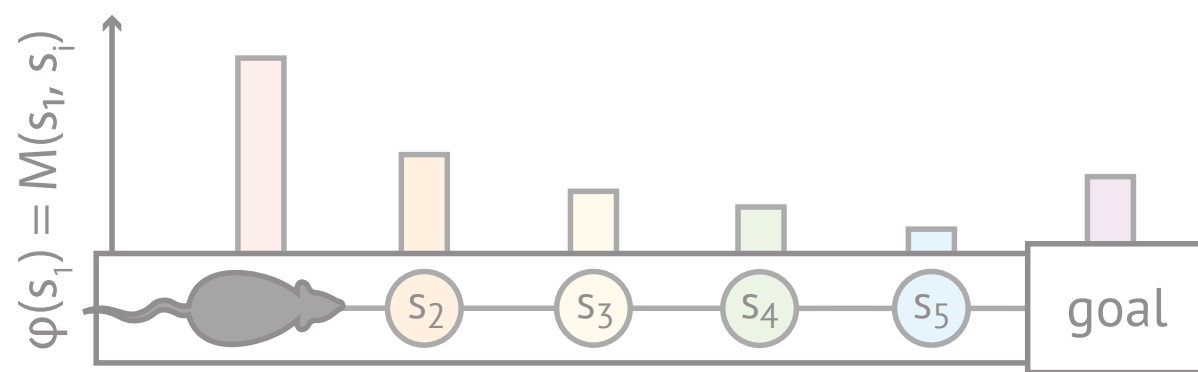
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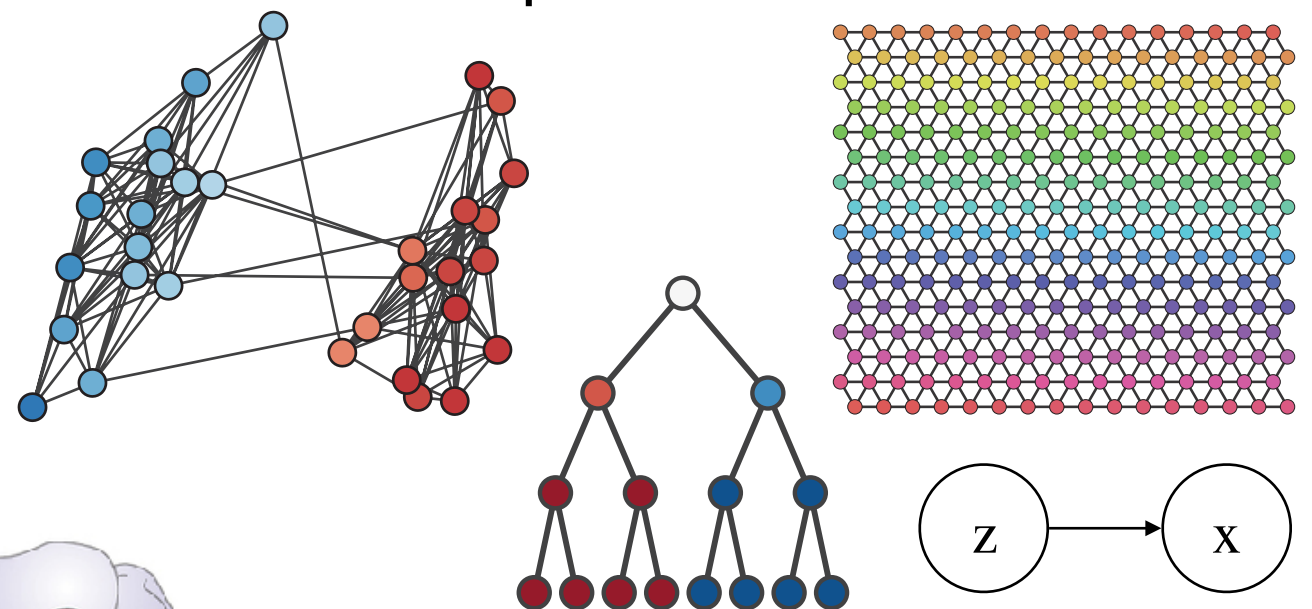
## Predictions

Represent states in terms of the predictions they make



## Compression

Learn as short description as possible



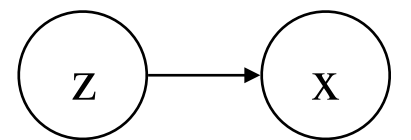
## Place Cells

For each state, learn how often you will visit other states?



## Grid Cells

Capture low-dimensional structure among predictions

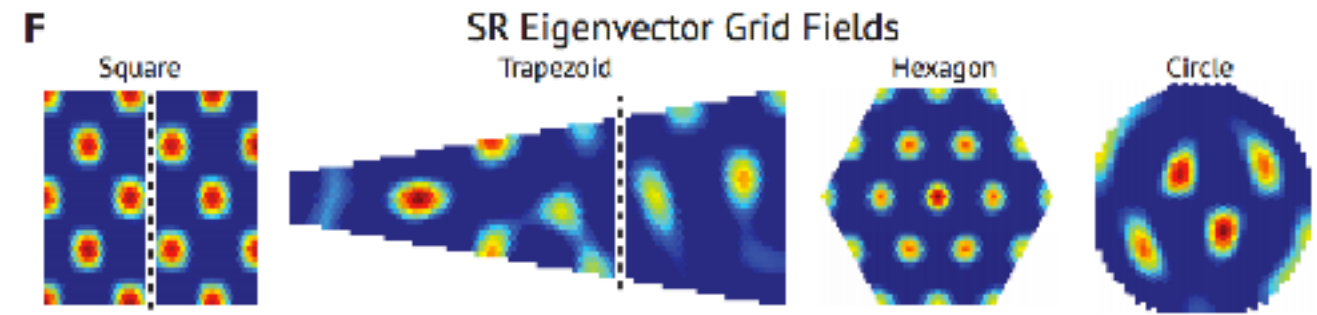




# Simulating Grid Cells

# Simulating Grid Cells

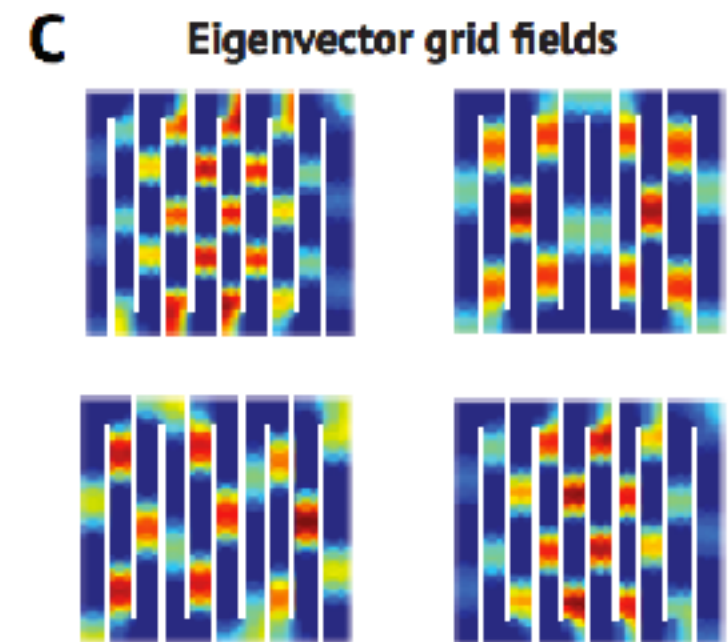
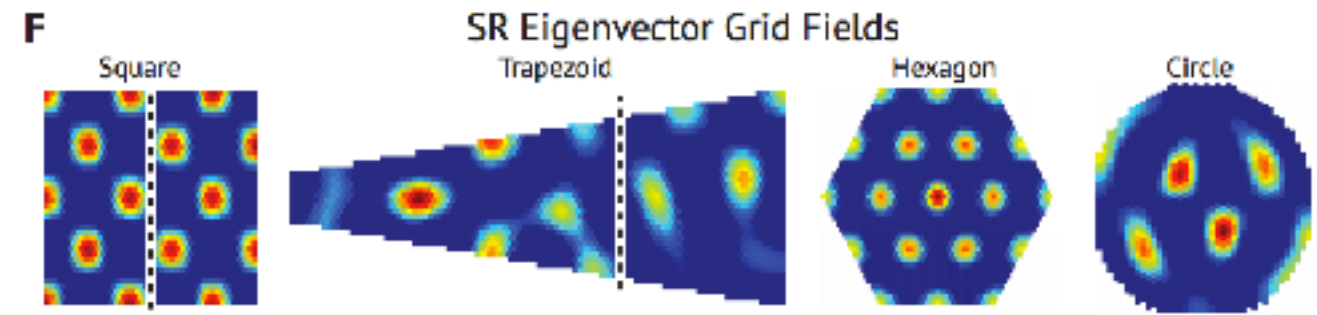
Eigenvectors of successor representation



# Simulating Grid Cells

Eigenvectors of successor representation

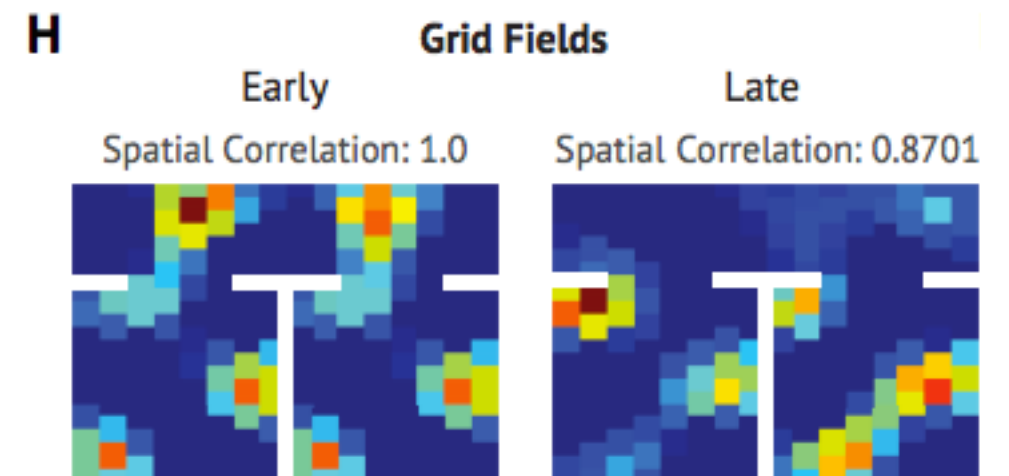
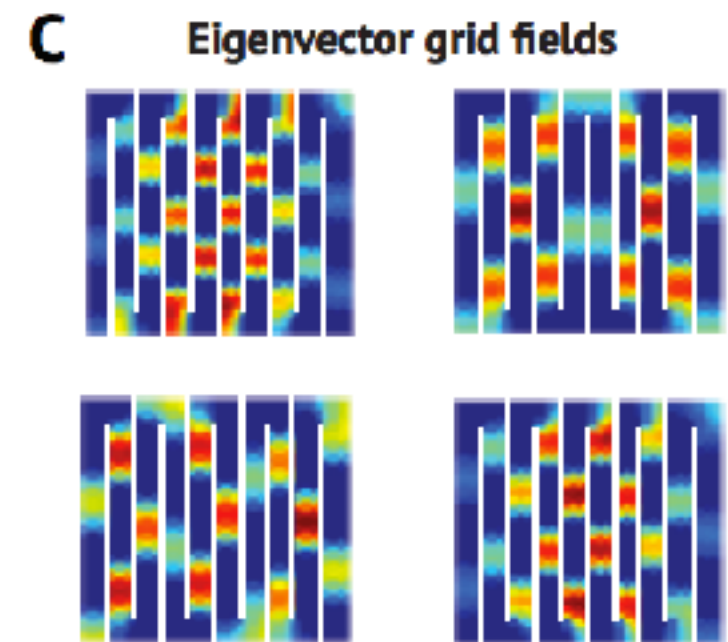
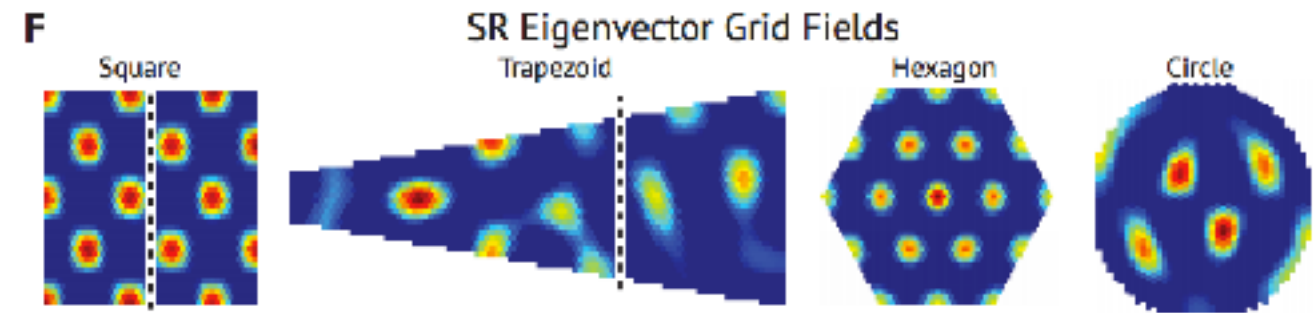
- Grid fields in geometric environments  
*Krupic et al. (2013)*



# Simulating Grid Cells

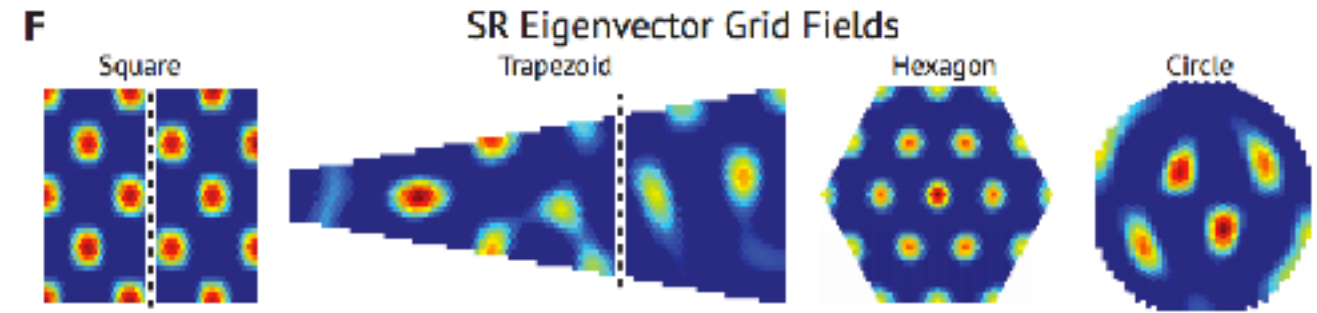
Eigenvectors of successor representation

- Grid fields in geometric environments  
*Krupic et al. (2013)*
- Fragmentation in Hairpin Maze  
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*Stachenfeld, Botvinick, Gershman (2017)*

# Simulating Grid Cells



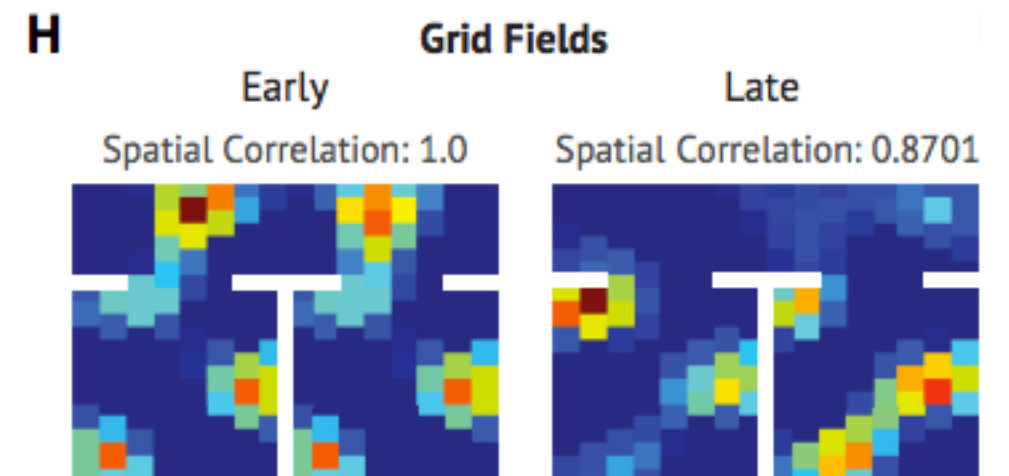
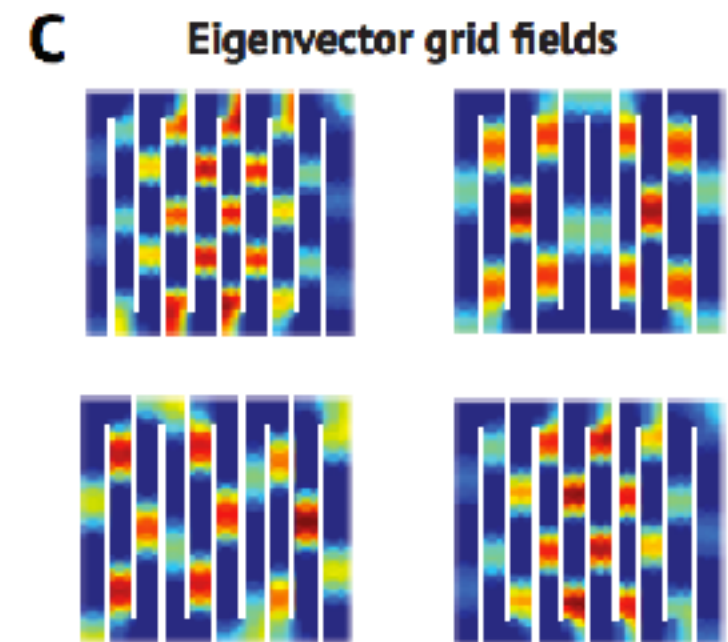
Eigenvectors of successor representation

- Grid fields in geometric environments  
*Krupic et al. (2013)*
- Fragmentation in Hairpin Maze  
*Derdikman et al. (2009)*
- Grid field learning in a multicompartment environment  
*Carpenter et al. (2017)*

**Takeaway:** we capture many aspects of how environmental geometry and topology are known to affect grid cells.

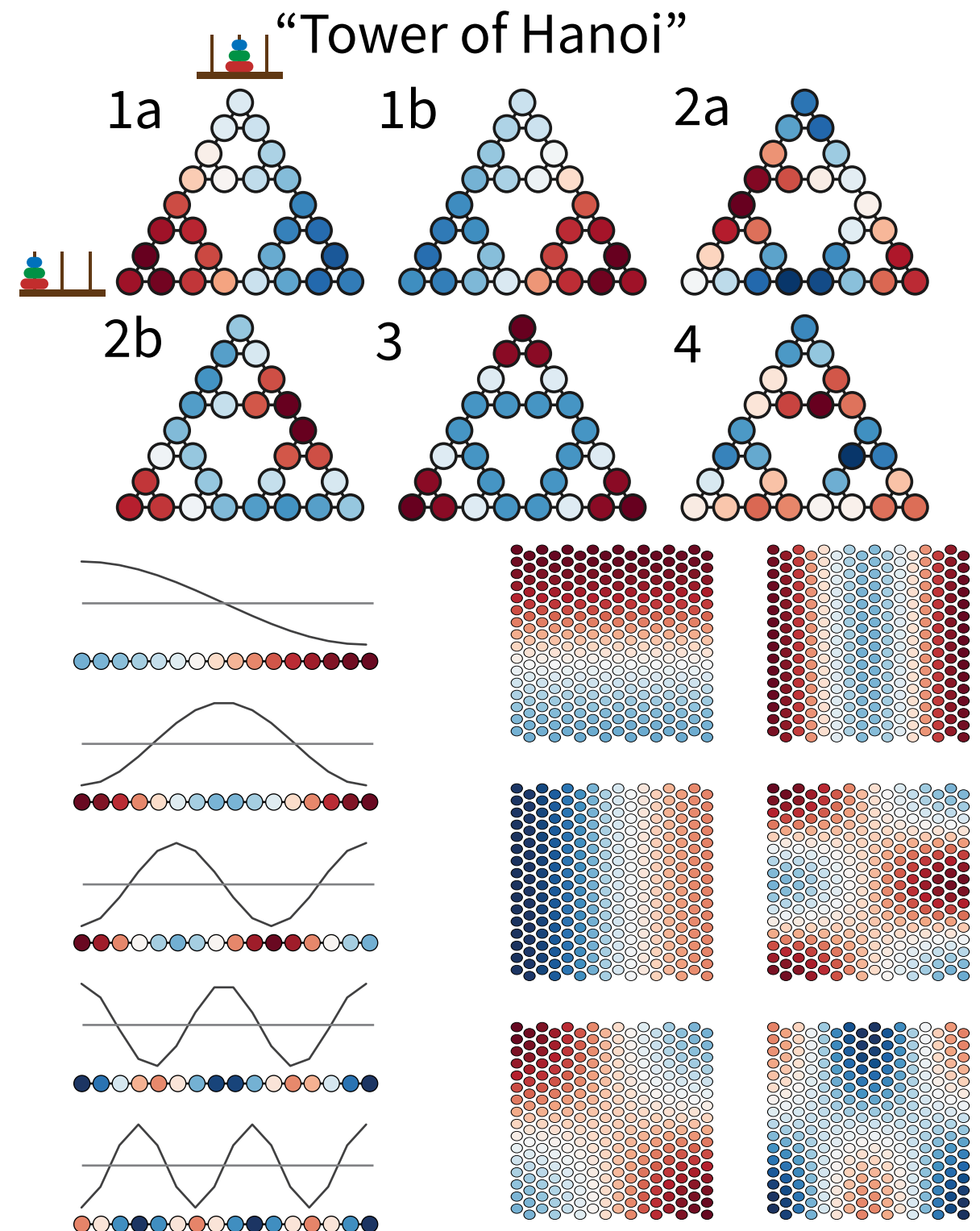
Related approach to other spectral methods, slow feature analysis

See also: Dordek et al 2016, Banino et al 2018, Sorscher et al 2019, Whittington et al 2019, Mok & Love 2019



*Stachenfeld, Botvinick, Gershman (2017)*

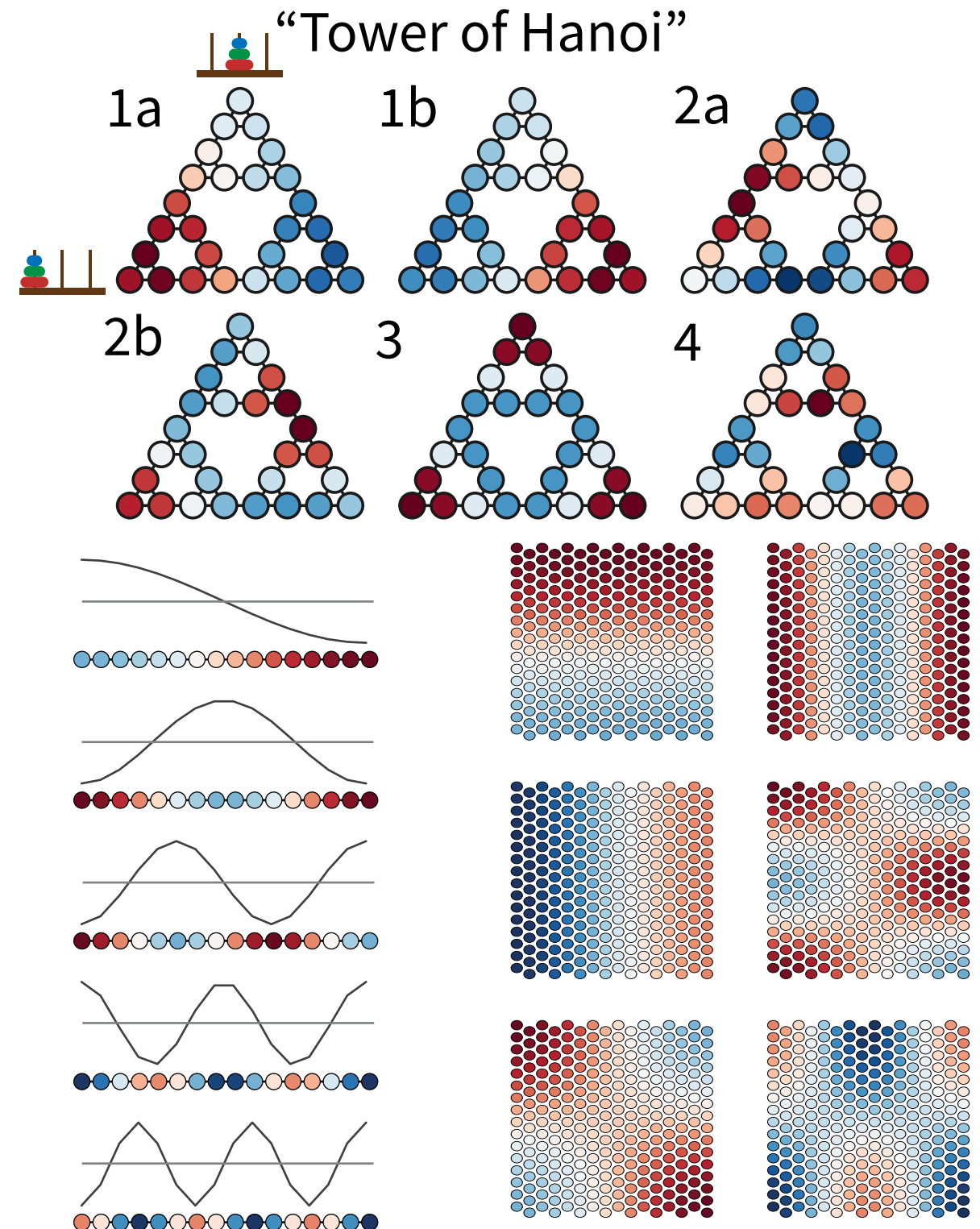
# Eigenvectors have a lot of nice applications





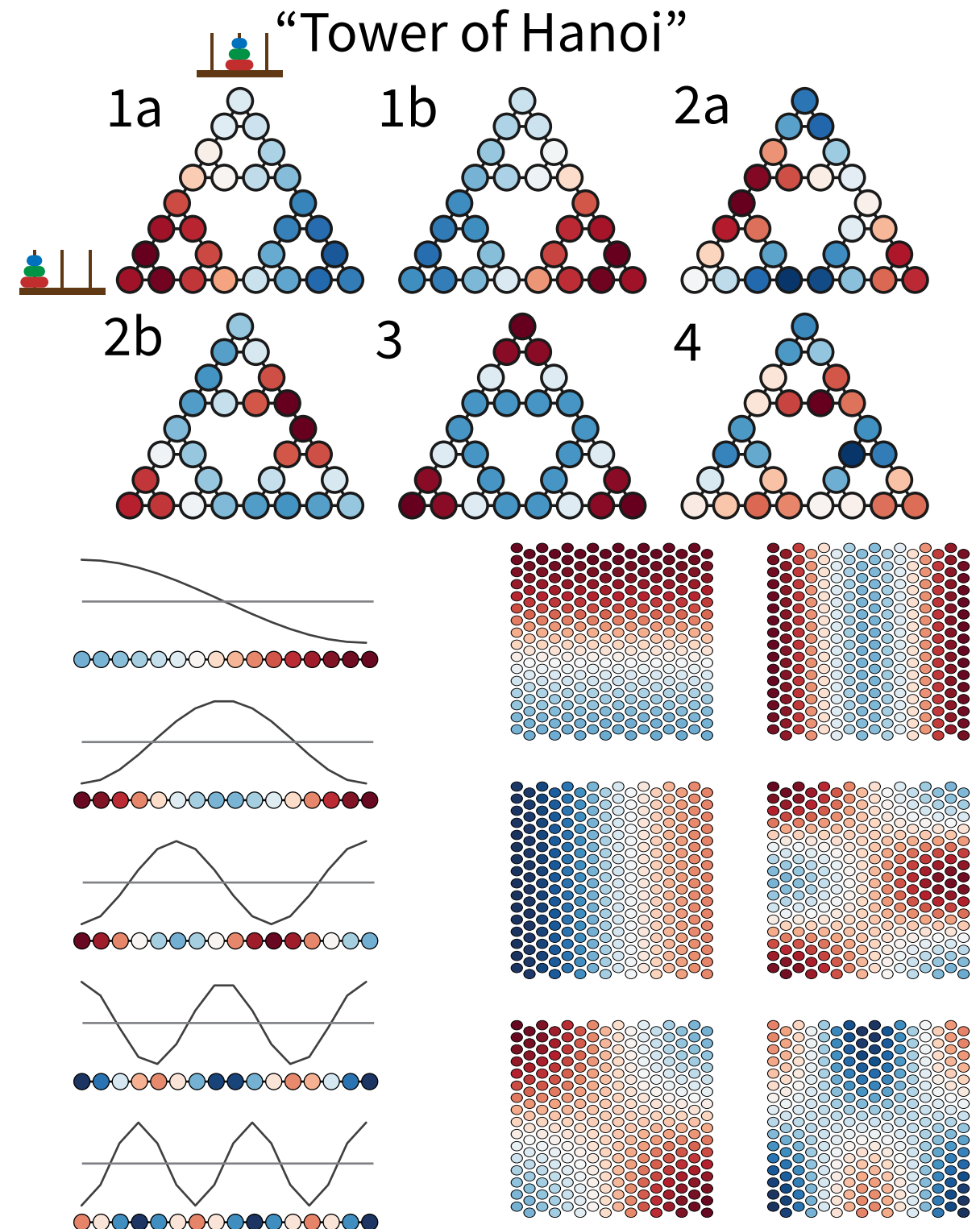
# Eigenvectors have a lot of nice applications

- For compression – sort of like PCA over transition structure



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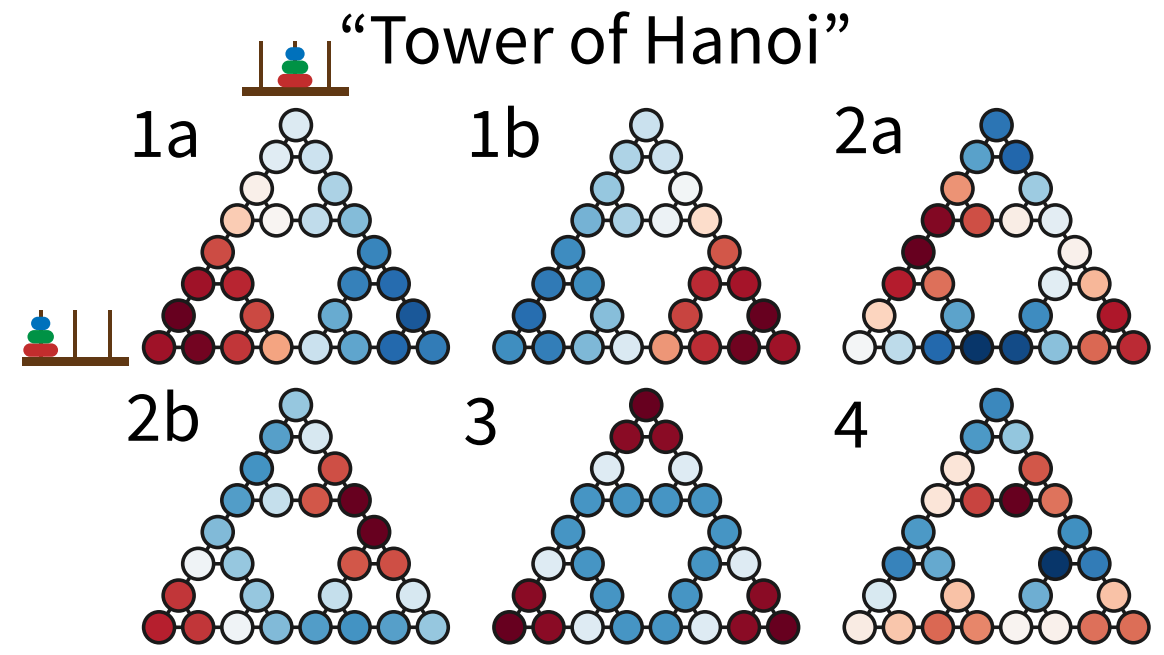
- For compression – sort of like PCA over transition structure
- Generalization of Fourier to graphs



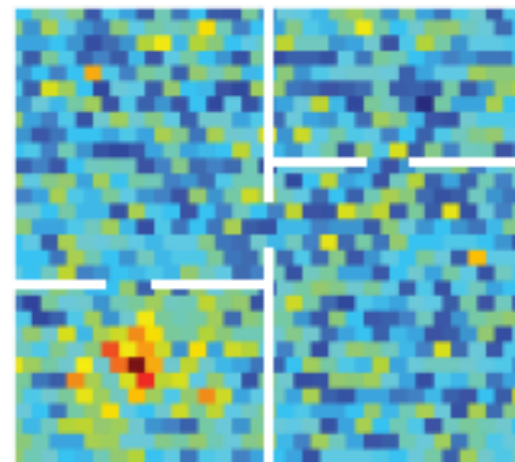


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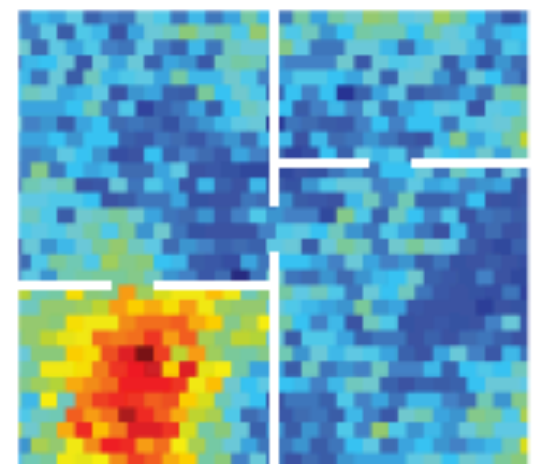
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- Noise correction + filling in missing data



**Nonlocal noise  
place field**

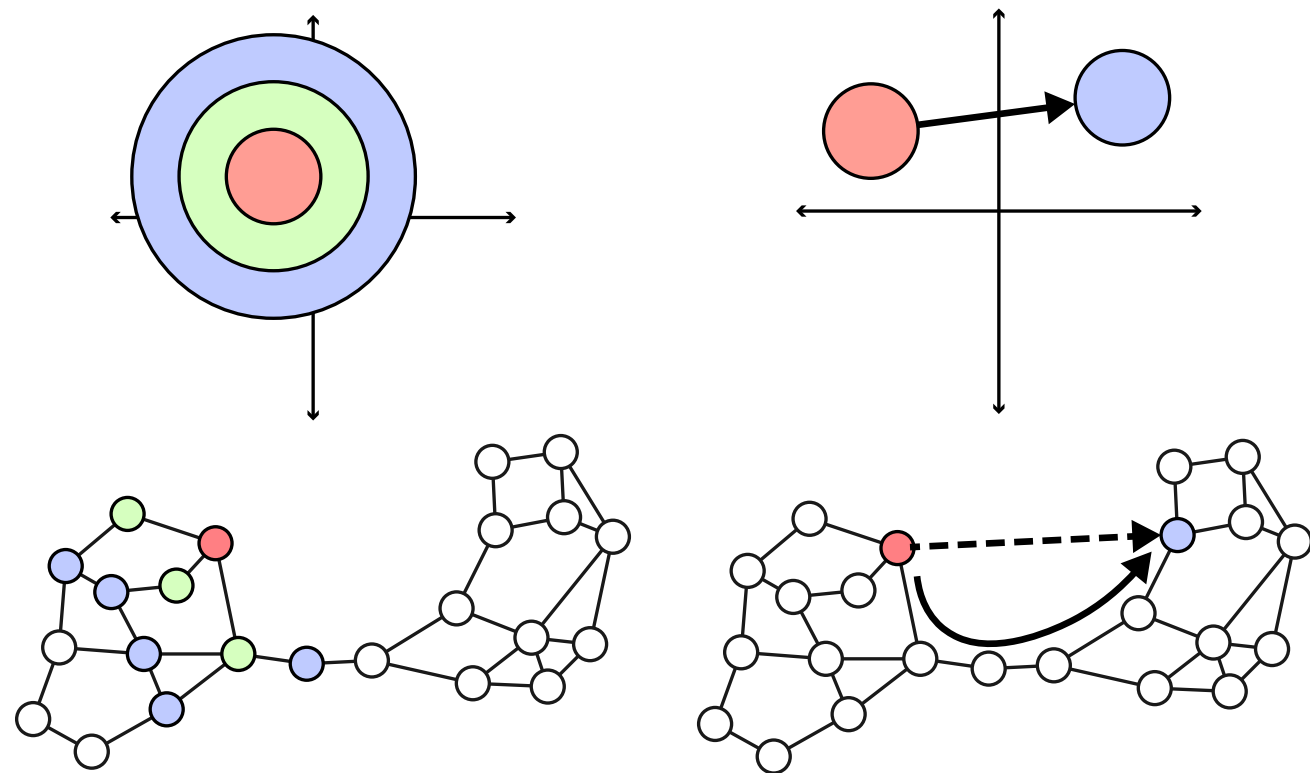
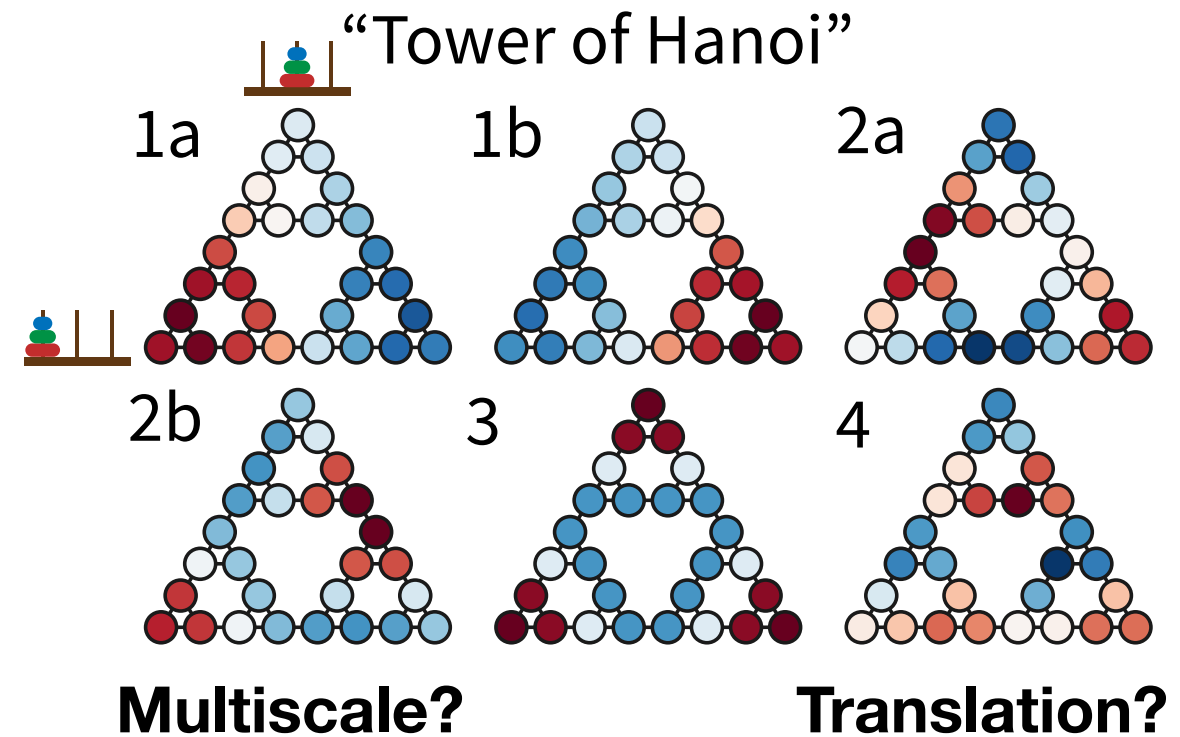


**Locally adjusted  
place field**



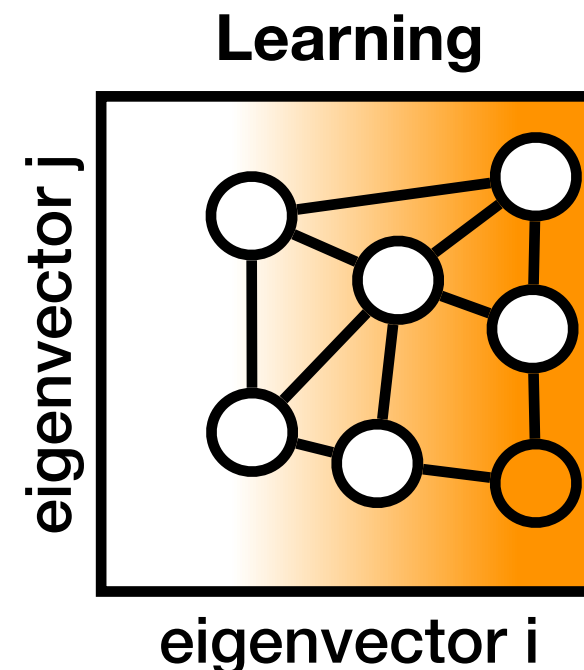
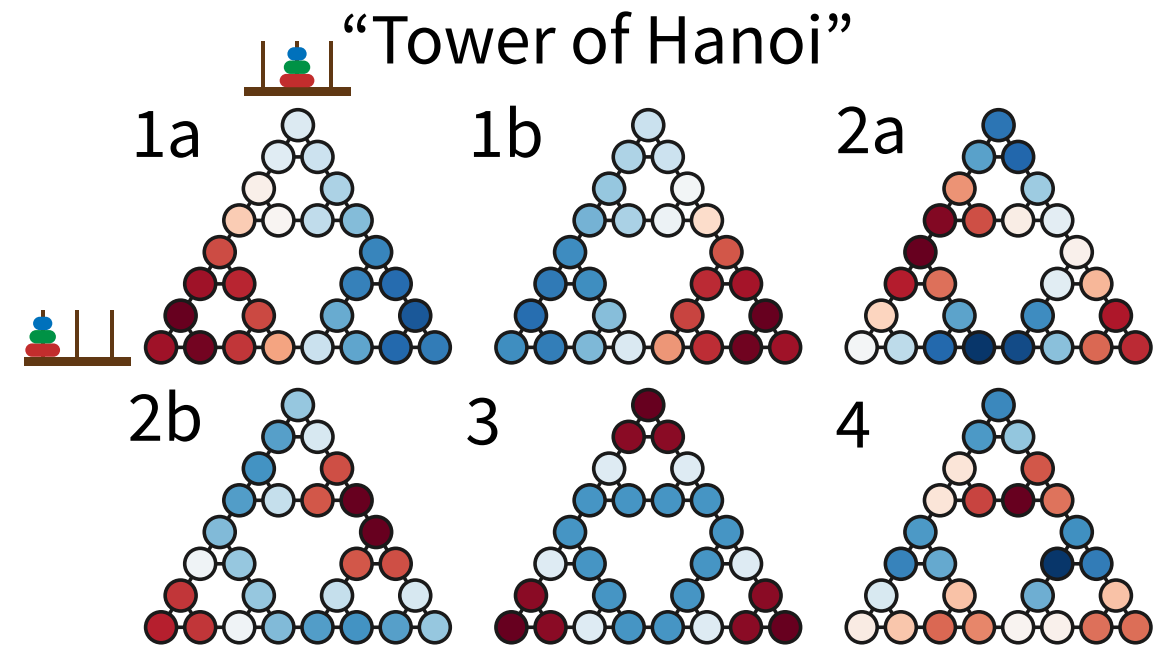
# Eigenvectors have a lot of nice applications

- For compression — sort of like PCA over transition structure
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- For “geometry on graphs” Generalize convolution and spatial features to arbitrary geometries



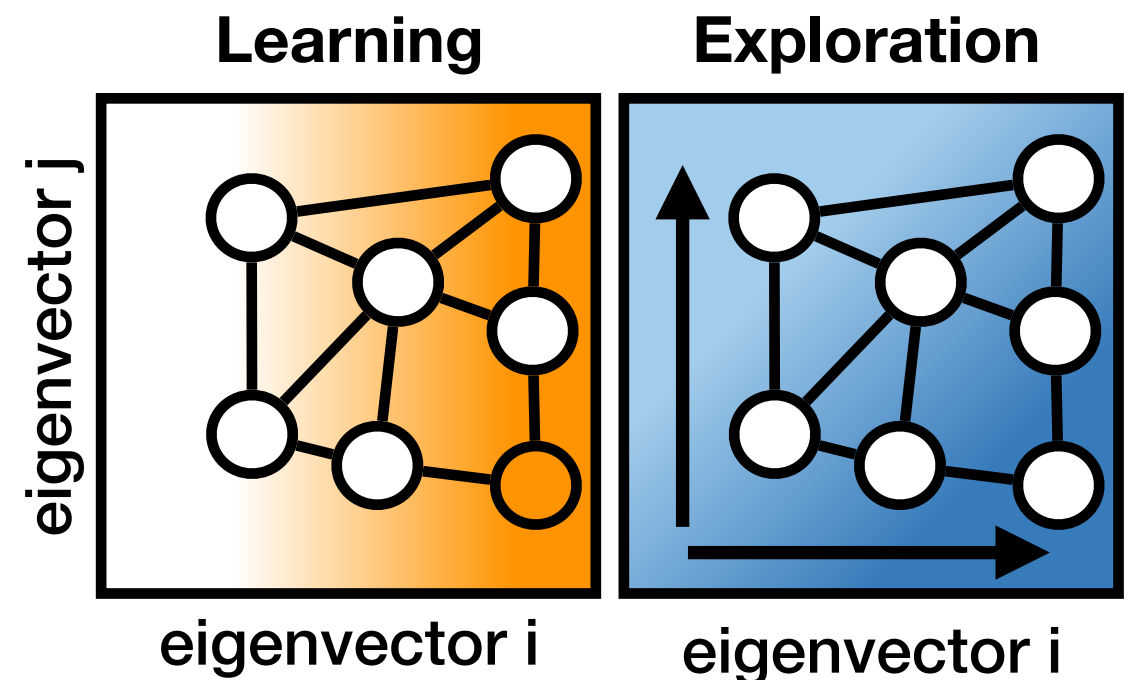
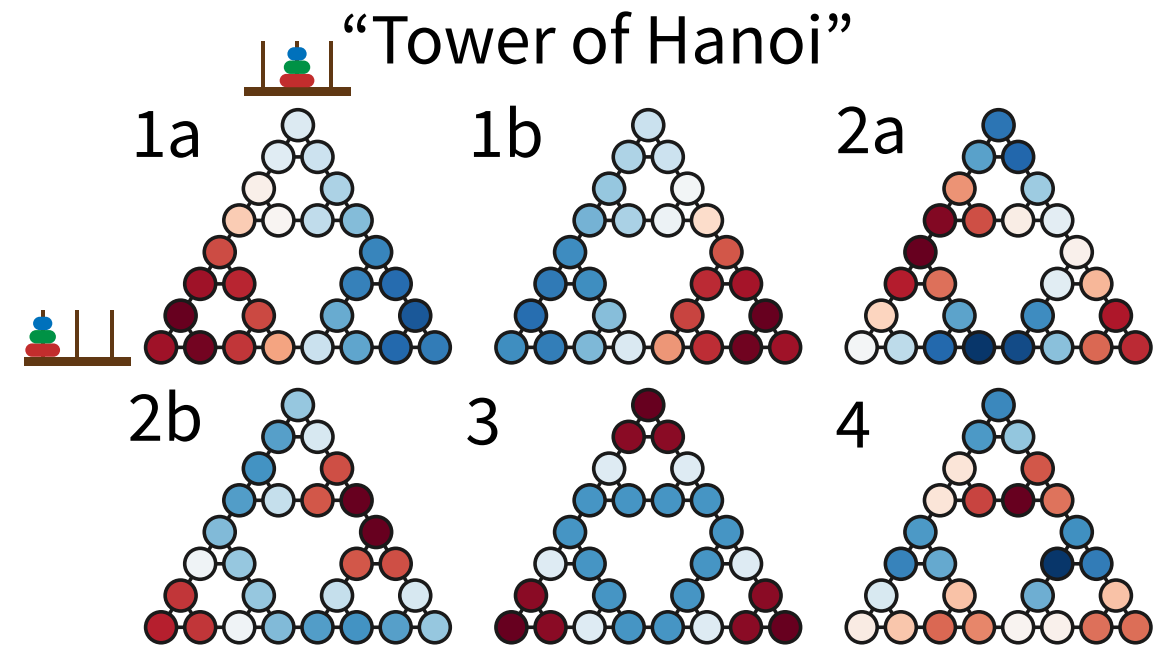
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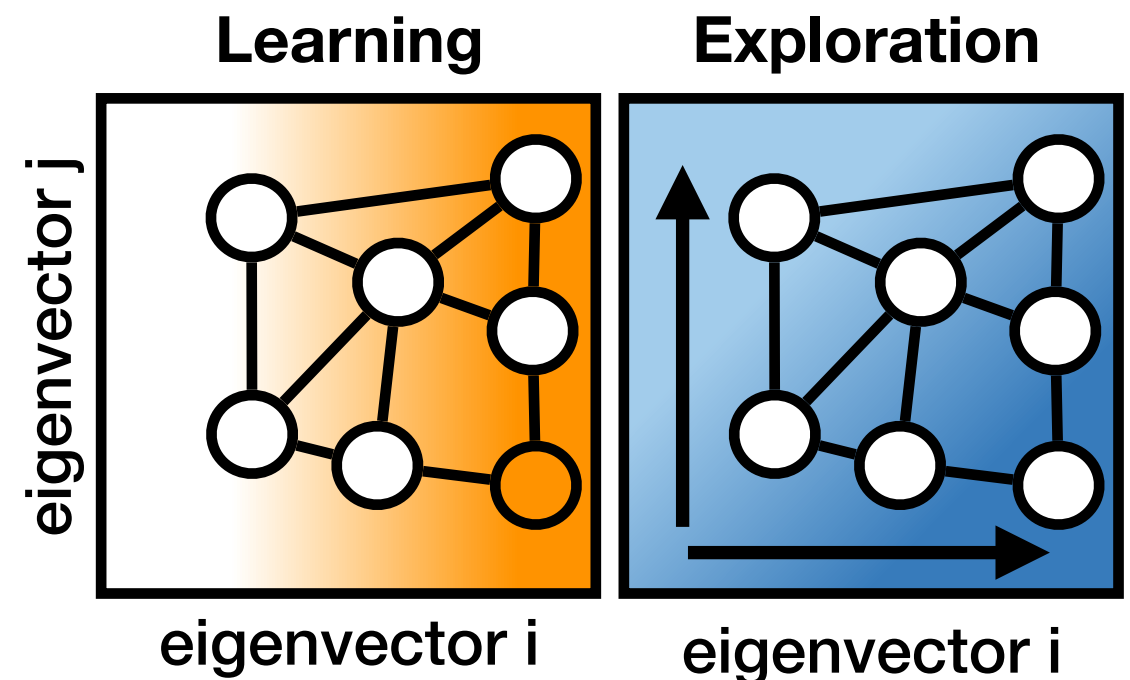
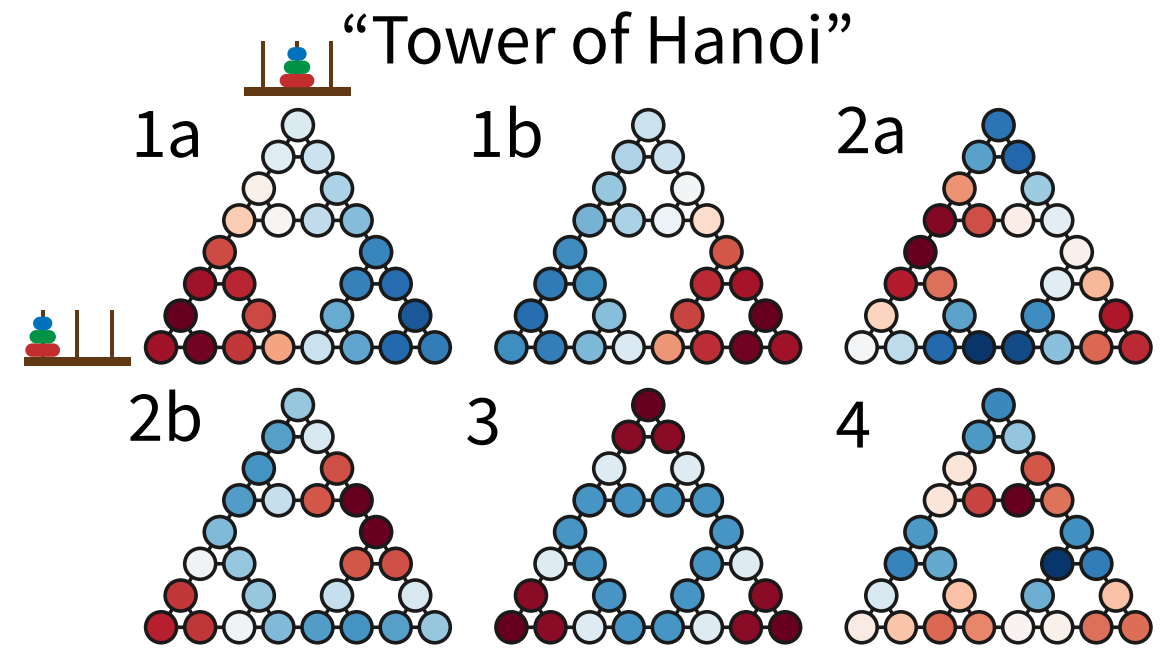
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- For supporting exploration: “Eigenoptions” Machado et al (2017a,b–2018)



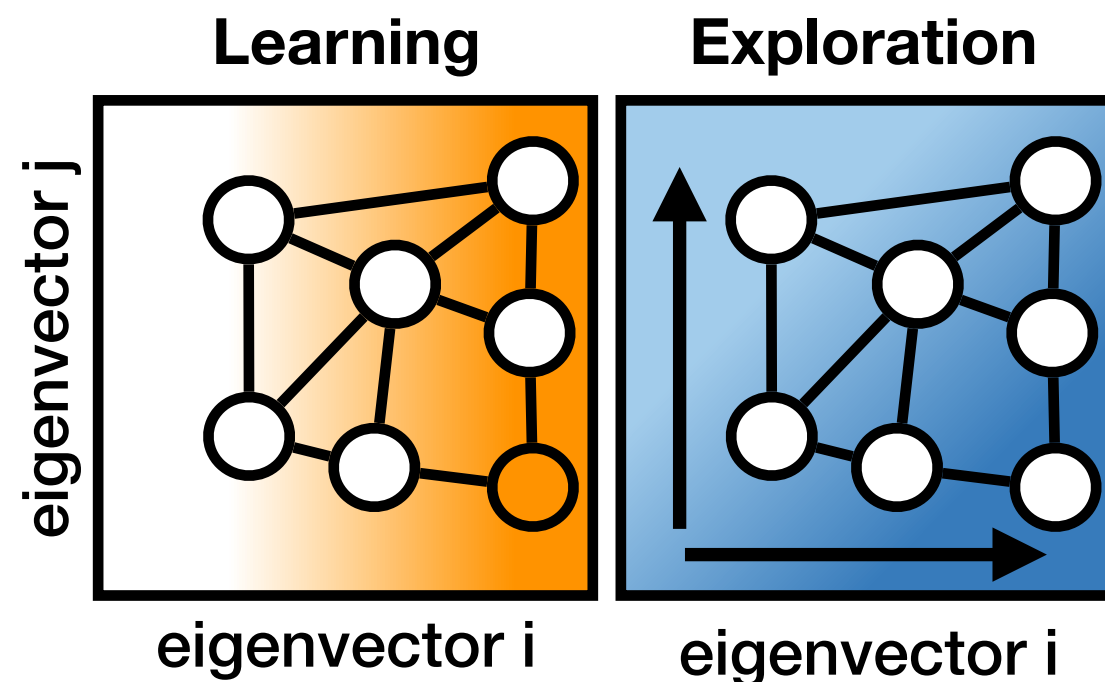
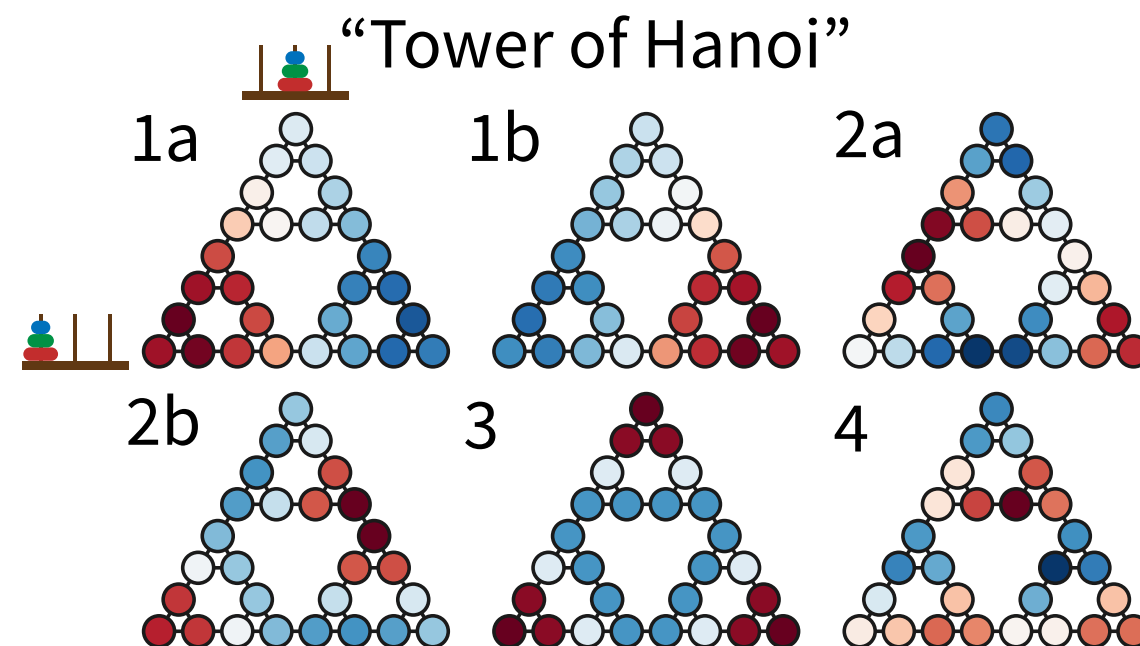
# Eigenvectors have a lot of nice applications

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- **For multiscale/hierarchical learning + exploration**



# Eigenvectors have a lot of nice applications

- **For compression** – sort of like PCA over transition structure
- **Generalization of Fourier to graphs**
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- **For supporting learning:** “Protovalue functions” (Mahadevan (2007))
- **For supporting exploration:** “Eigenoptions” Machado et al (2017a,b–2018)
- **For multiscale/hierarchical learning + exploration**
- **Graph Convolutional Neural Nets** Bruna et al 2013



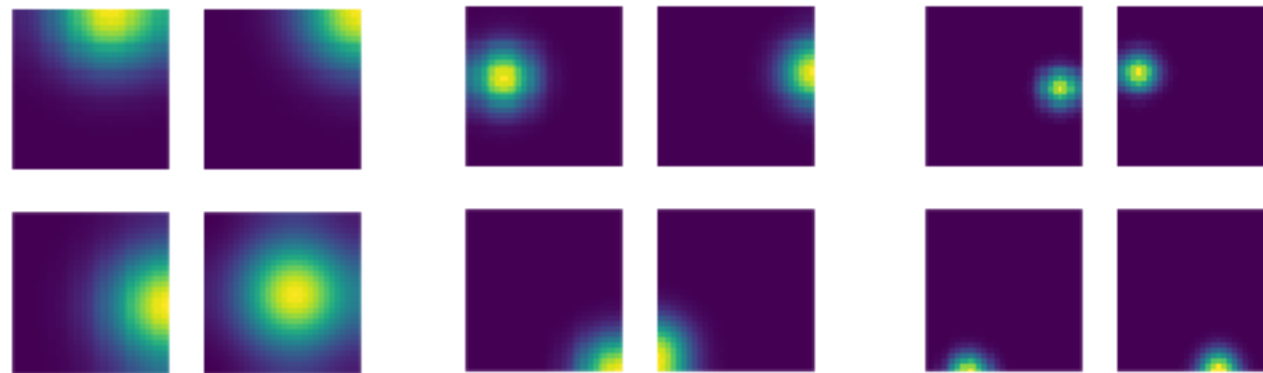


# Hierarchical Learning + Exploration

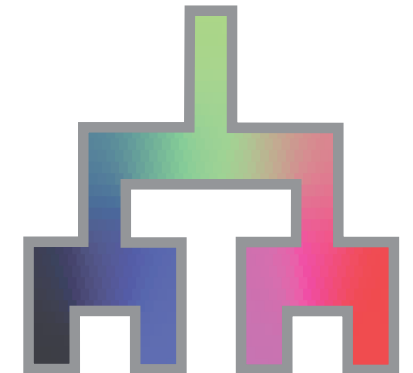
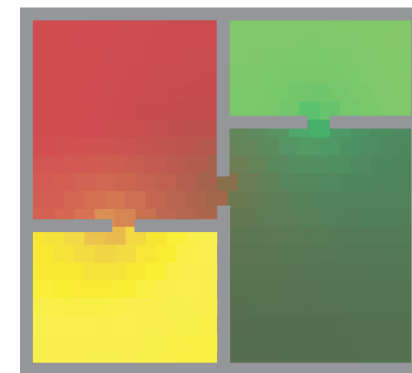
Dan  
McNamee



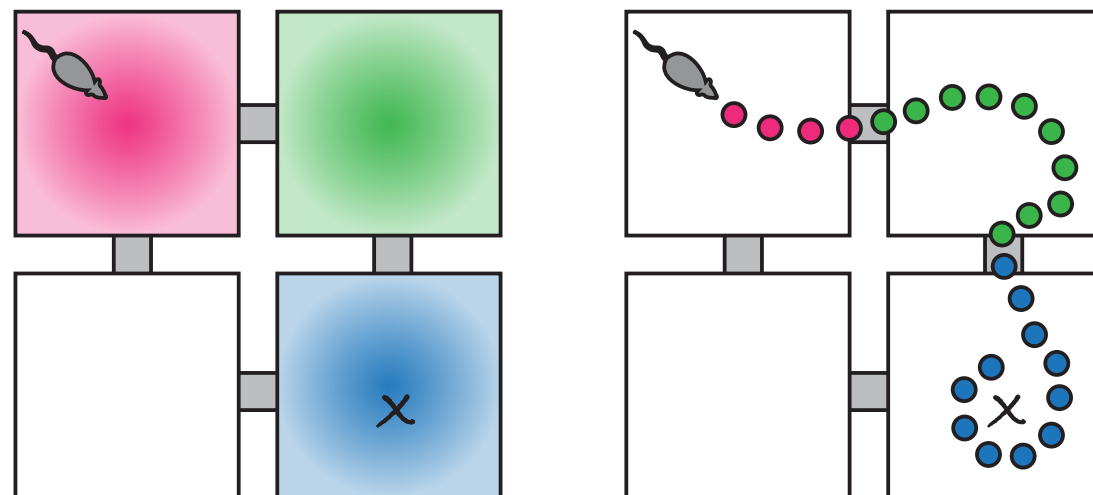
**Multi-scale**



**Detect structural boundaries**



**Supports hierarchical learning + planning**



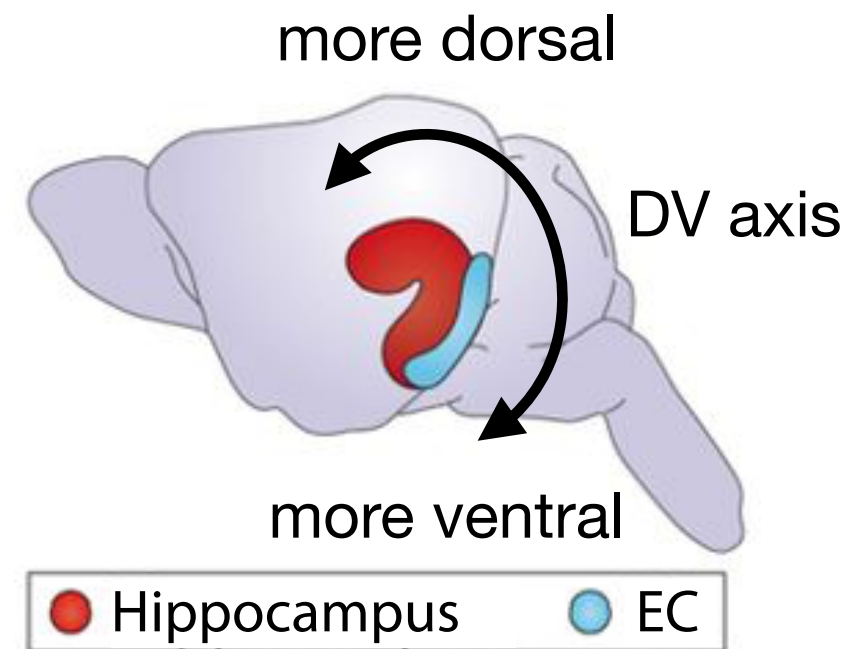
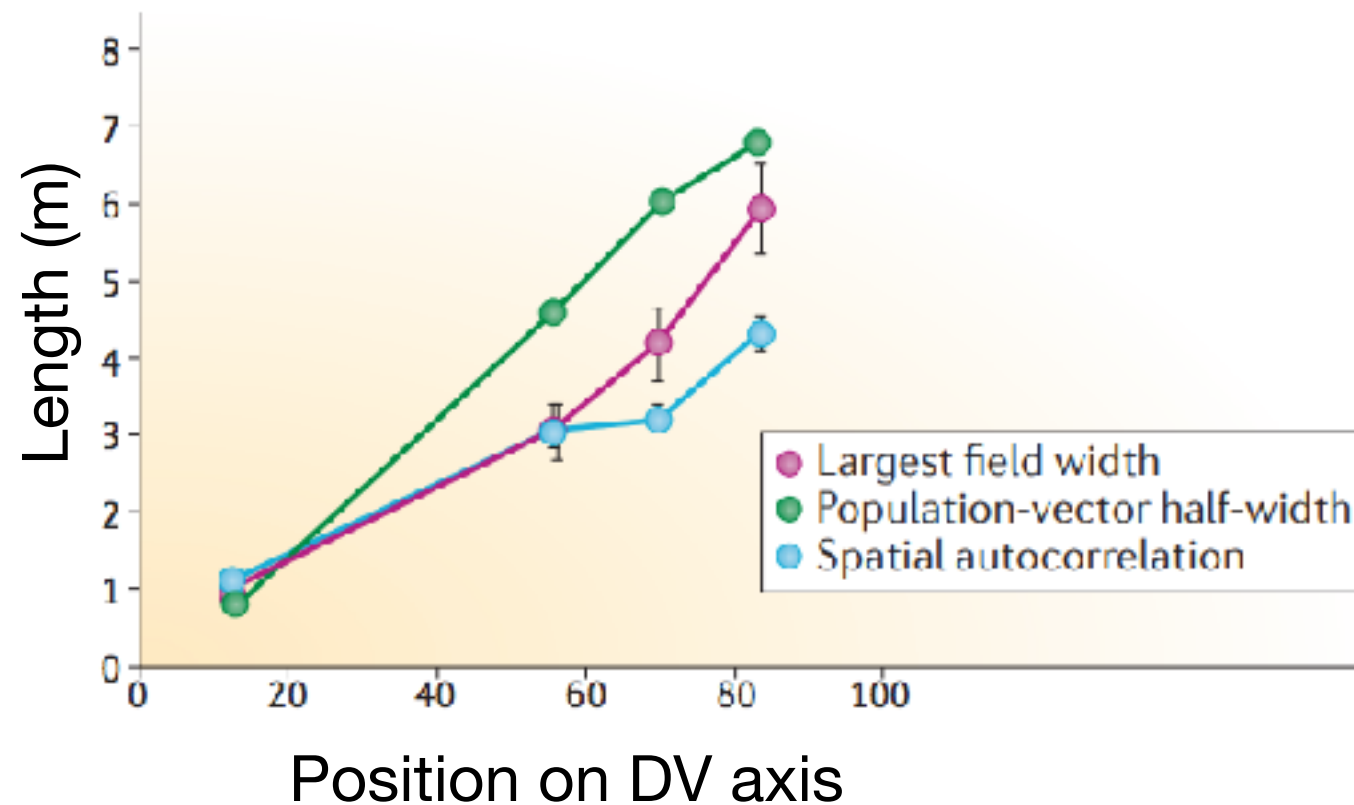
High Level

Low Level



# Multi-scale representations

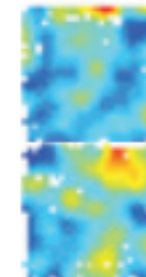
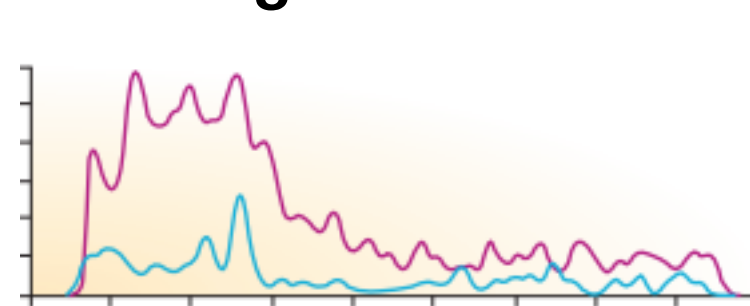
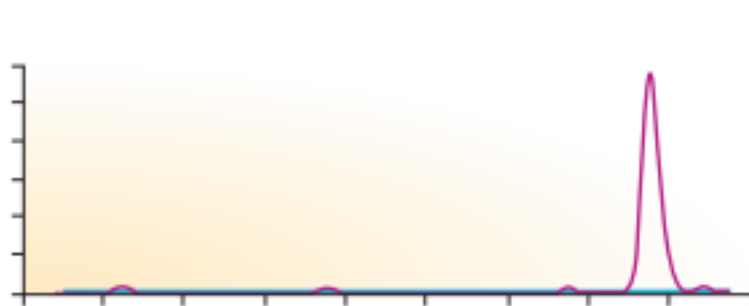
Range of spatial scales along longitudinal axis of  
hippocampus + MEC



Small dorsal field

Large ventral field

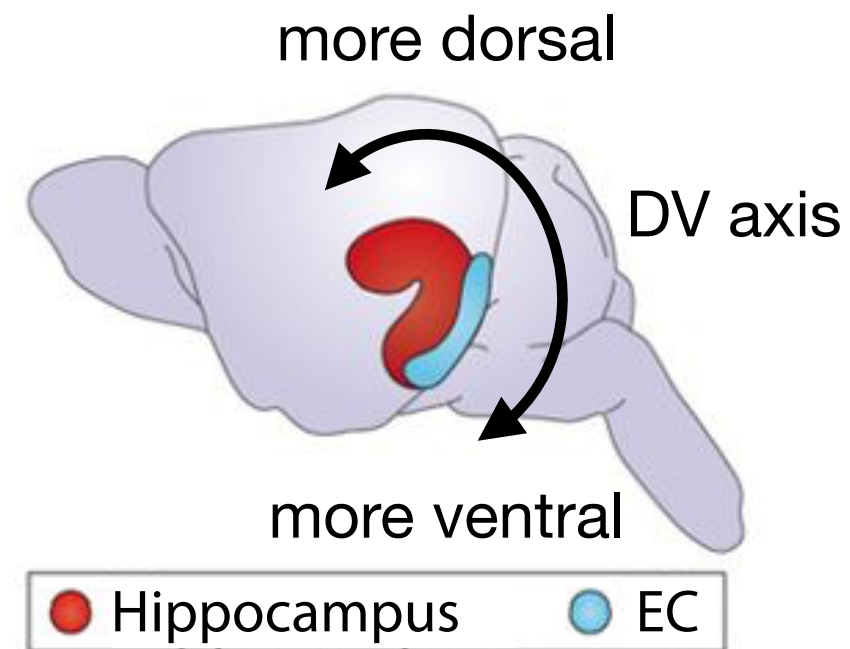
Firing Rate



*Review by Strange et al (2014)*

# Multi-scale representations

Same eigenvectors support different scales

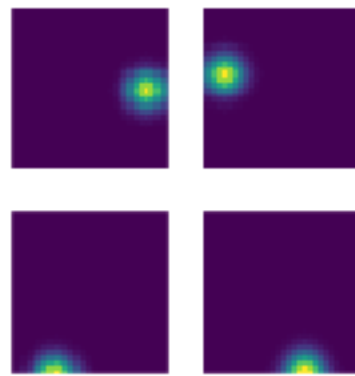


*Stachenfeld, Botvinick, Gershman (2017)  
also see Momennejad & Howard (2018) for things you can do with multi-scale SRs*

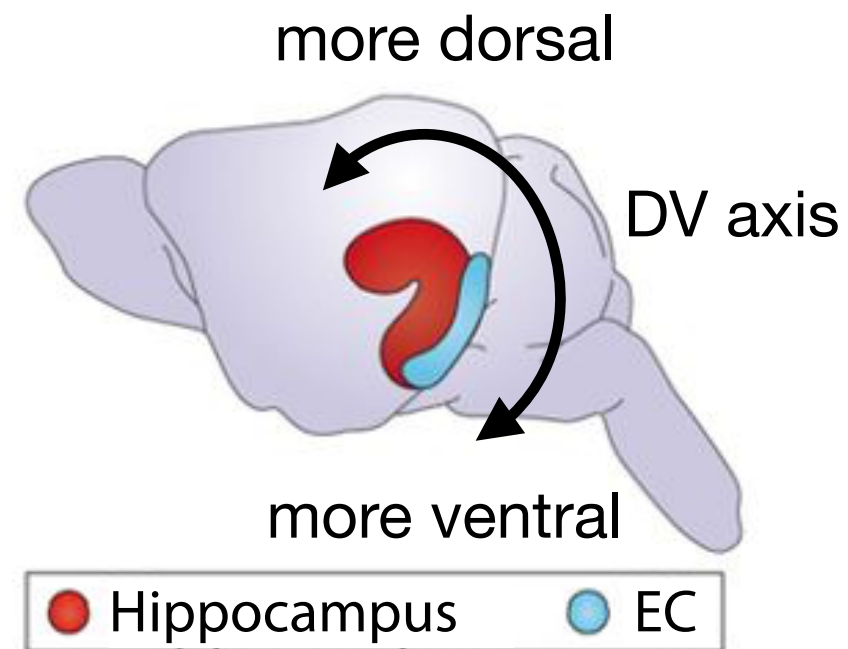
# Multi-scale representations

Same eigenvectors support different scales

**Simulated place fields with  
different scales**



**more dorsal**  
smaller discount factor

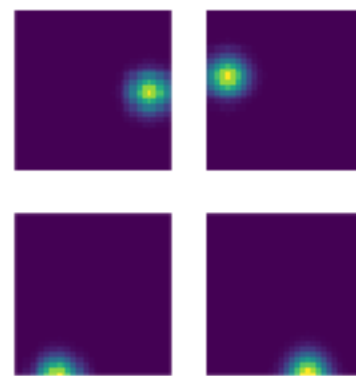


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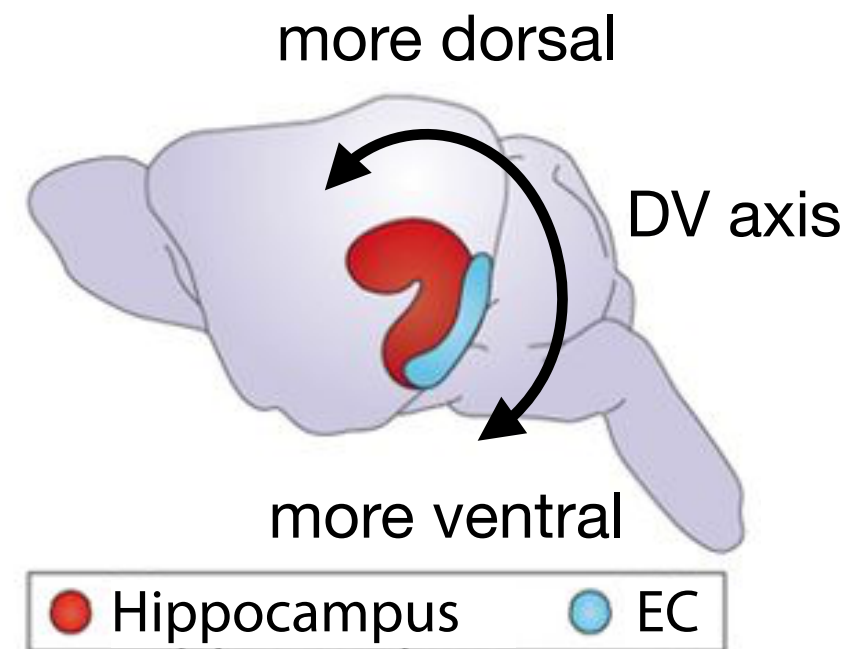
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**Simulated eigenvector grid fields**



more ventral

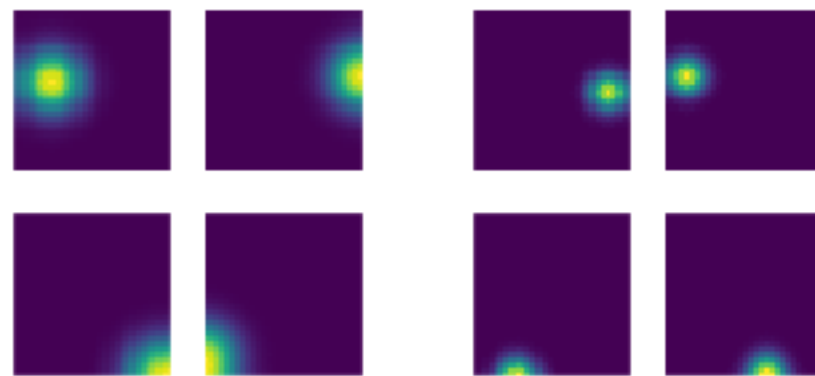
more dorsal

Stachenfeld, Botvinick, Gershman (2017)  
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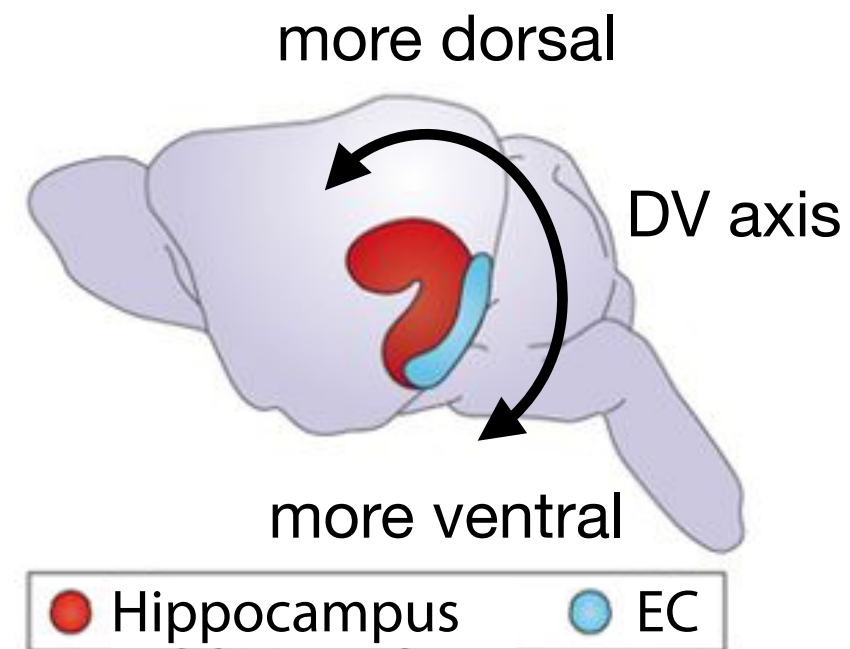
# Multi-scale representations

Same eigenvectors support different scales

**Simulated place fields with different scales**



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smaller discount factor



**Simulated eigenvector grid fields**



**more ventral**

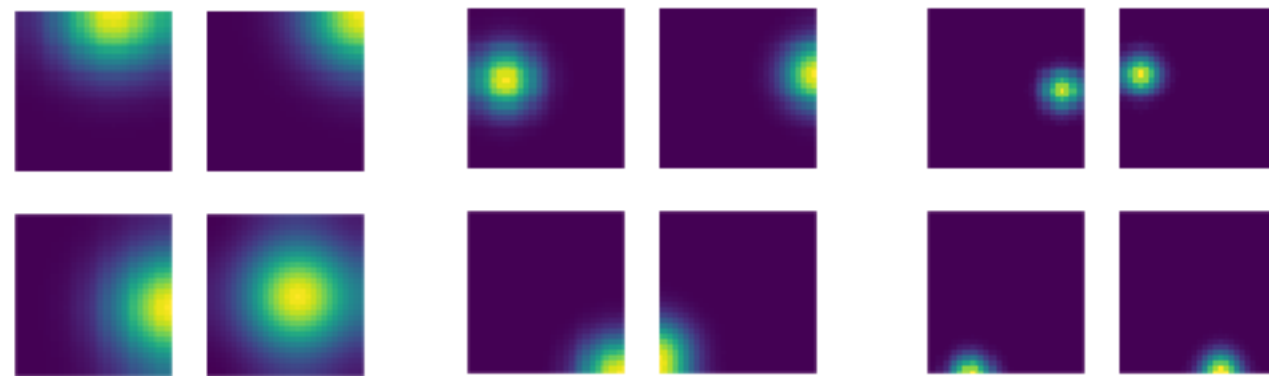
**more dorsal**

*Stachenfeld, Botvinick, Gershman (2017)*  
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# Multi-scale representations

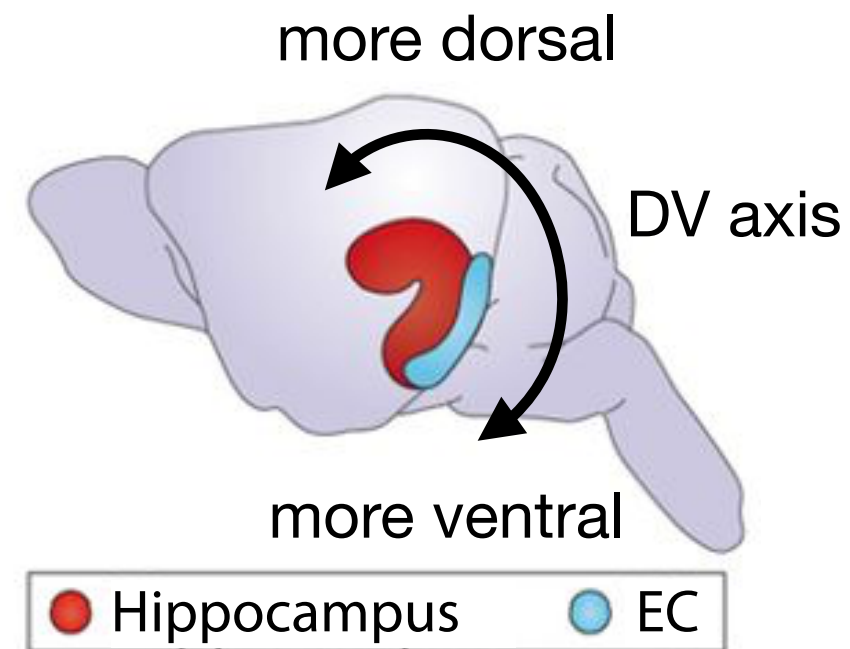
Same eigenvectors support different scales

**Simulated place fields with different scales**



**more ventral**  
larger discount factor

**more dorsal**  
smaller discount factor



**Simulated eigenvector grid fields**



**more ventral**

**more dorsal**

Stachenfeld, Botvinick, Gershman (2017)  
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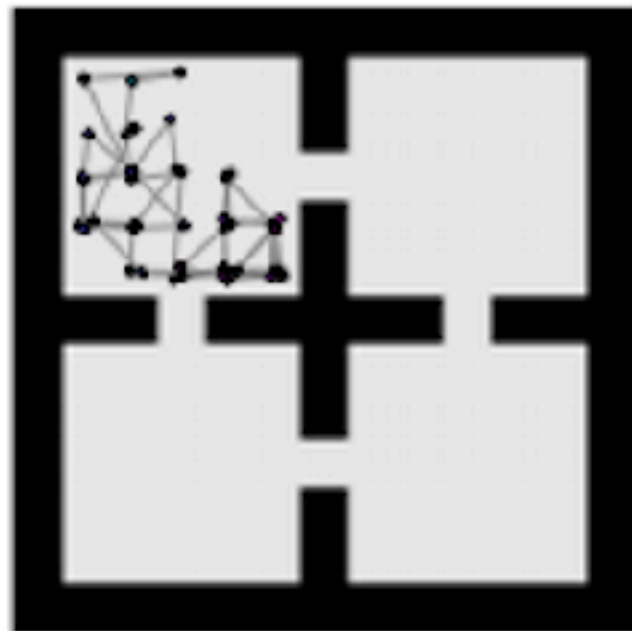
# Sample with different statistics

Dan  
McNamee



## Random Walk

Agent gets  
stuck in room





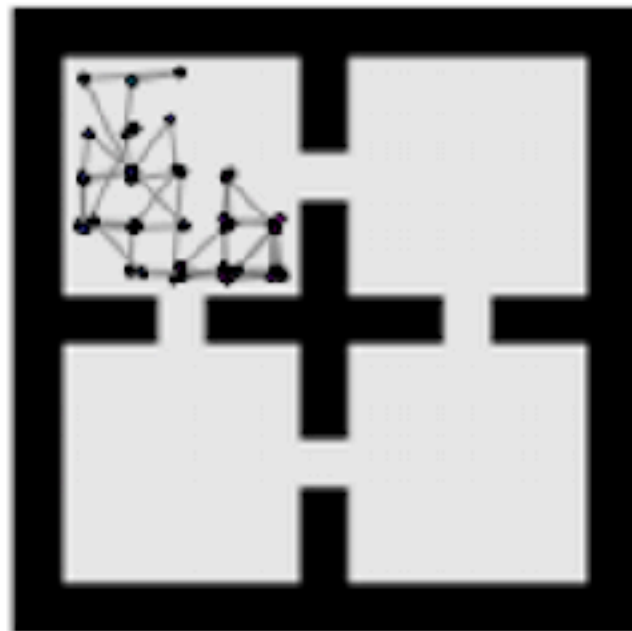
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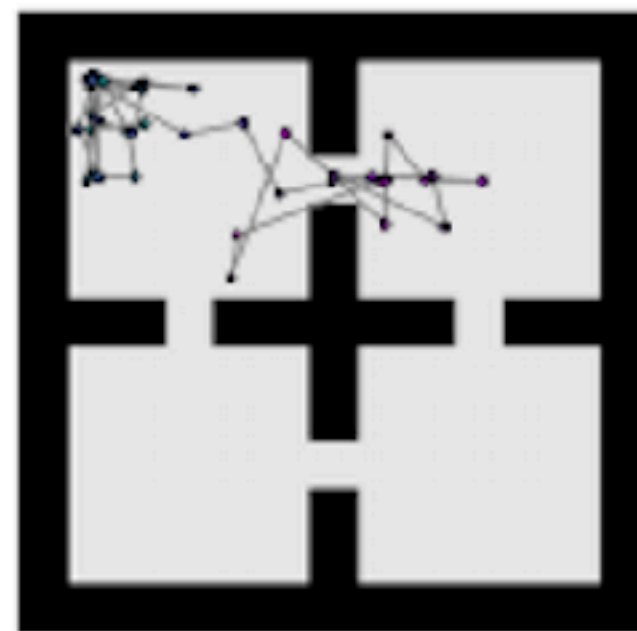
**Random Walk**

**Agent gets  
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**Levy Walk**

**Agent frees itself  
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faster**



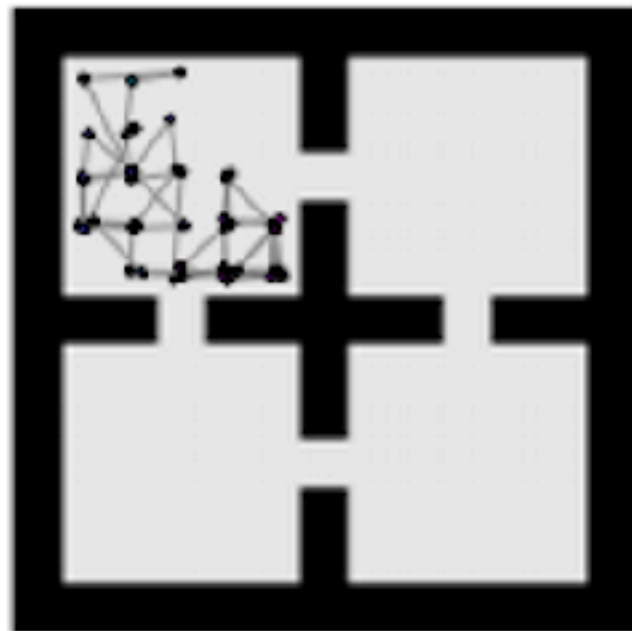
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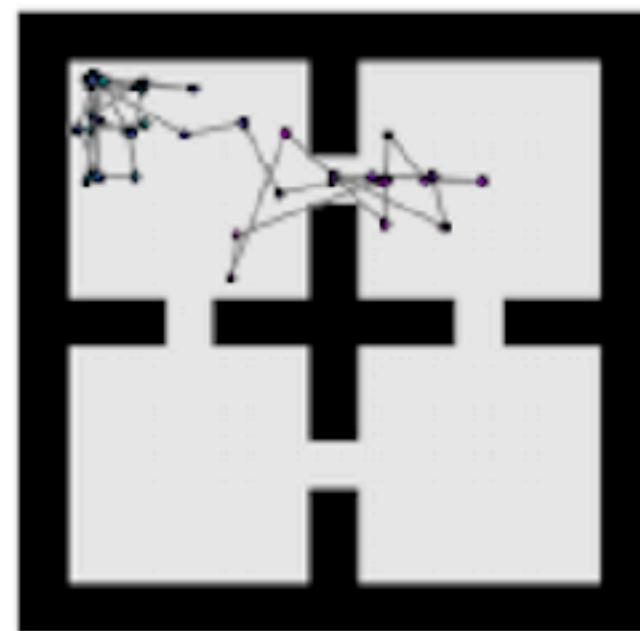
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**Levy walks experienced  
during foraging behavior**

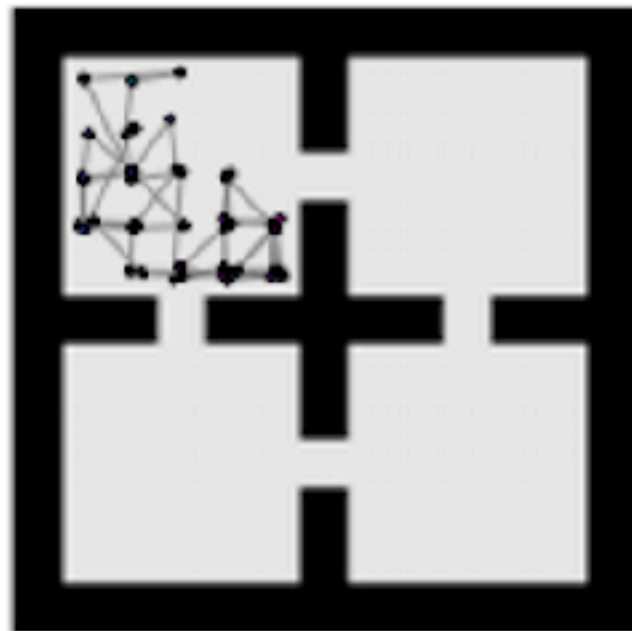
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Dan  
McNamee



**Random Walk**

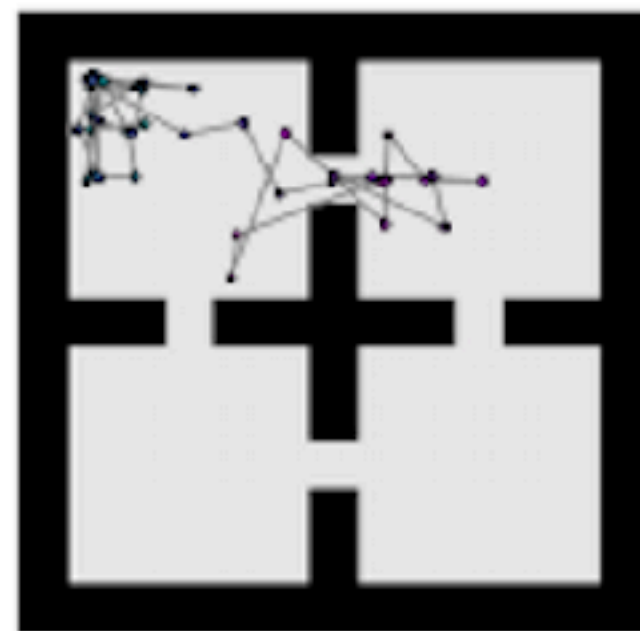
**Agent gets  
stuck in room**



**Random walk  
observed during sleep**

**Levy Walk**

**Agent frees itself  
from room, allowing  
it to find reward  
faster**

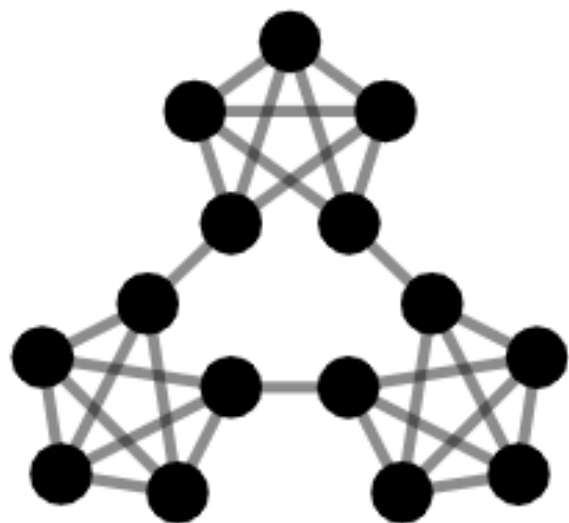


**Levy walks experienced  
during foraging behavior**

Stella et al (2019)

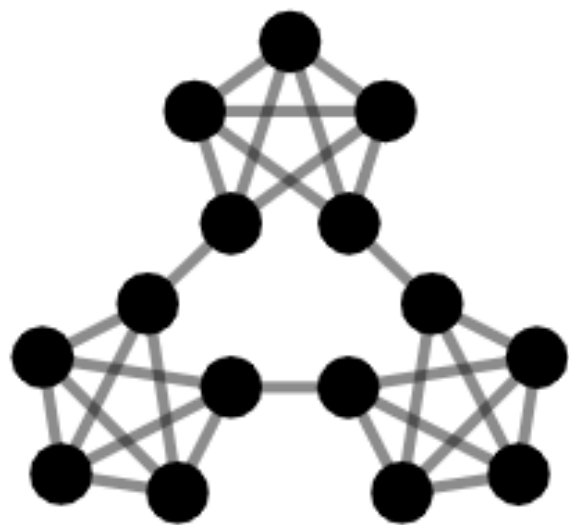
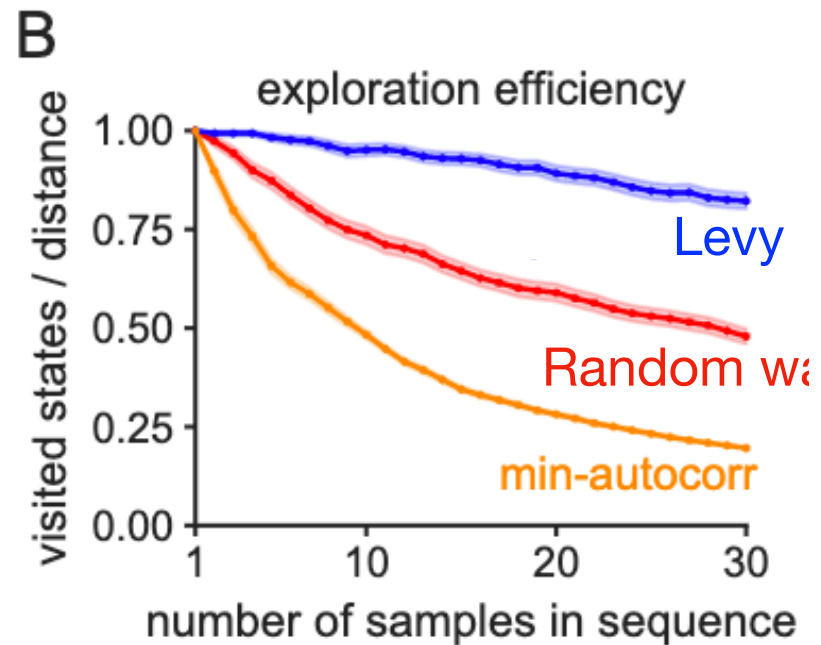
# Different advantages for different sampling modes

Dan  
McNamee



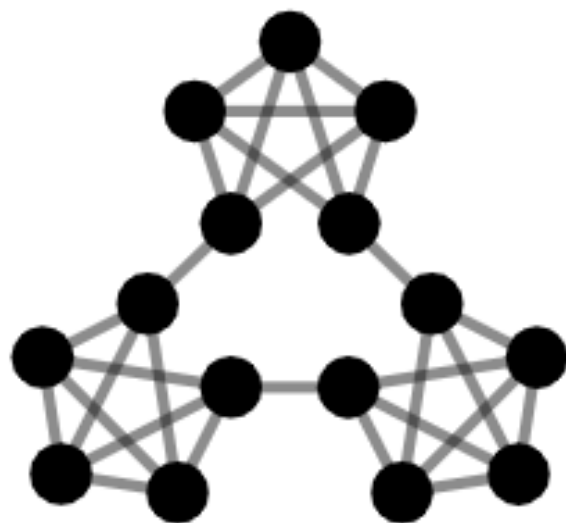
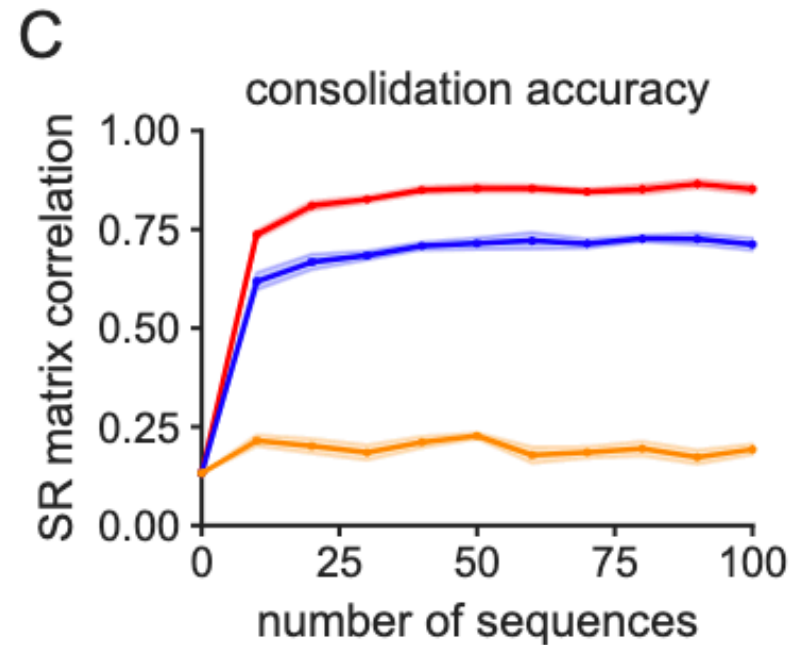
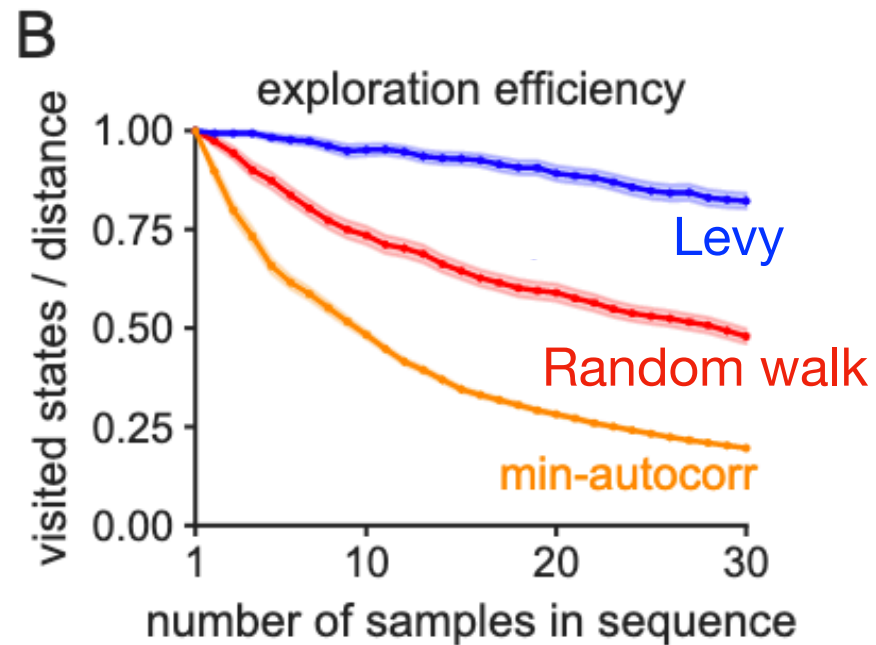
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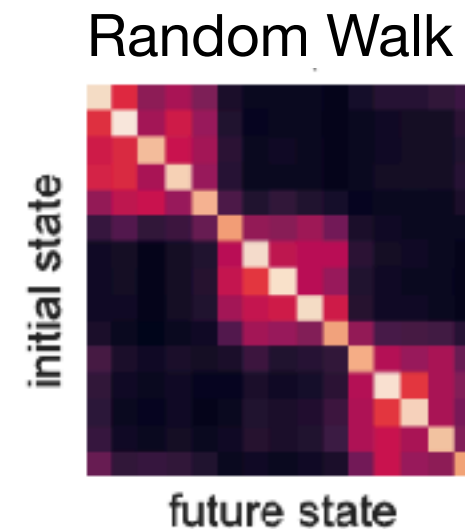
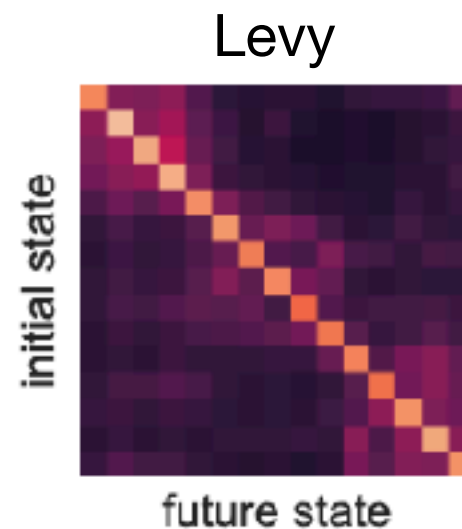
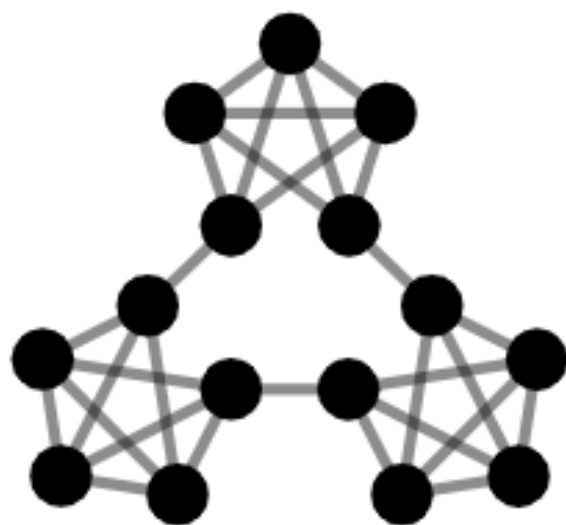
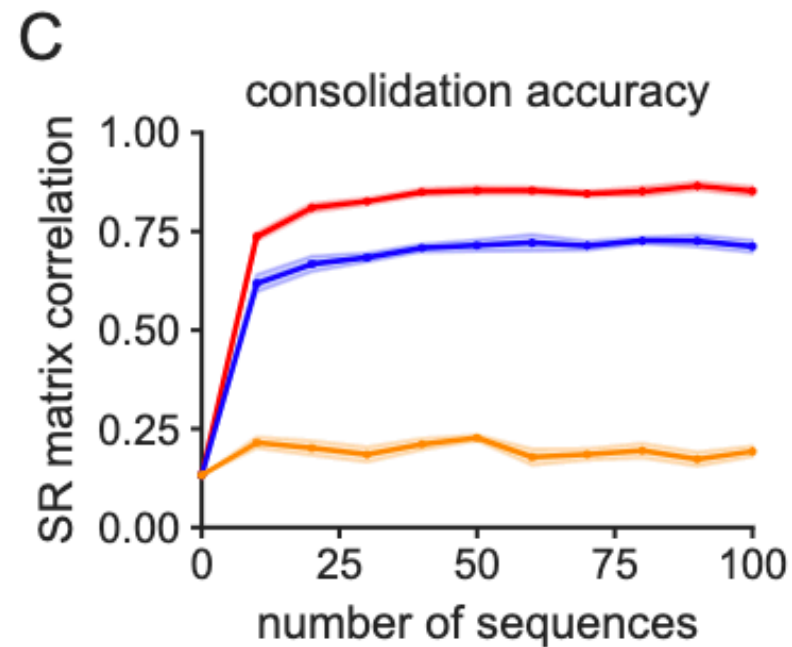
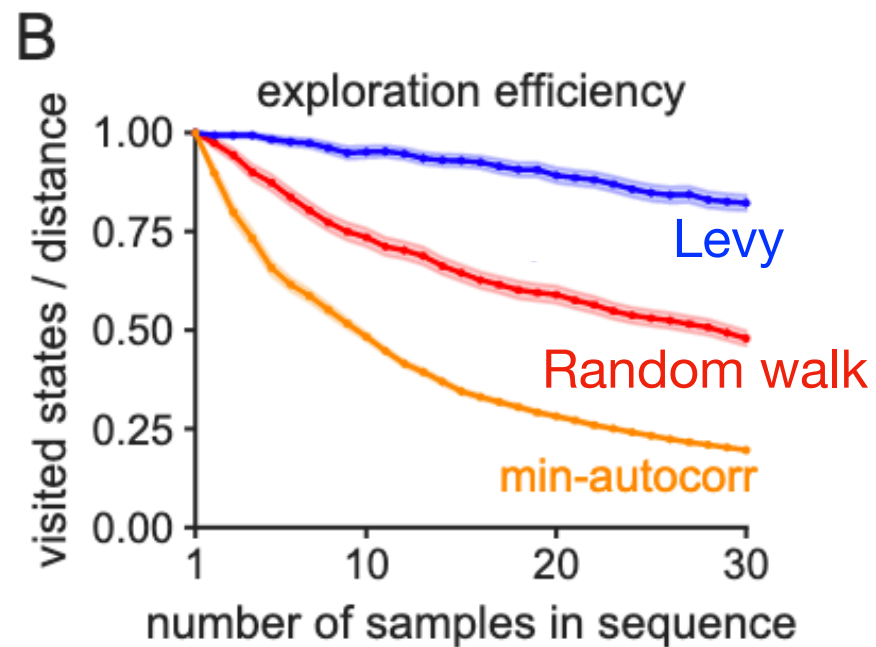
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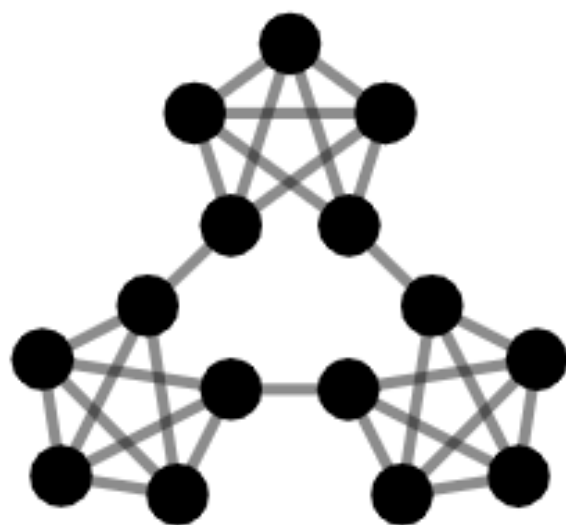
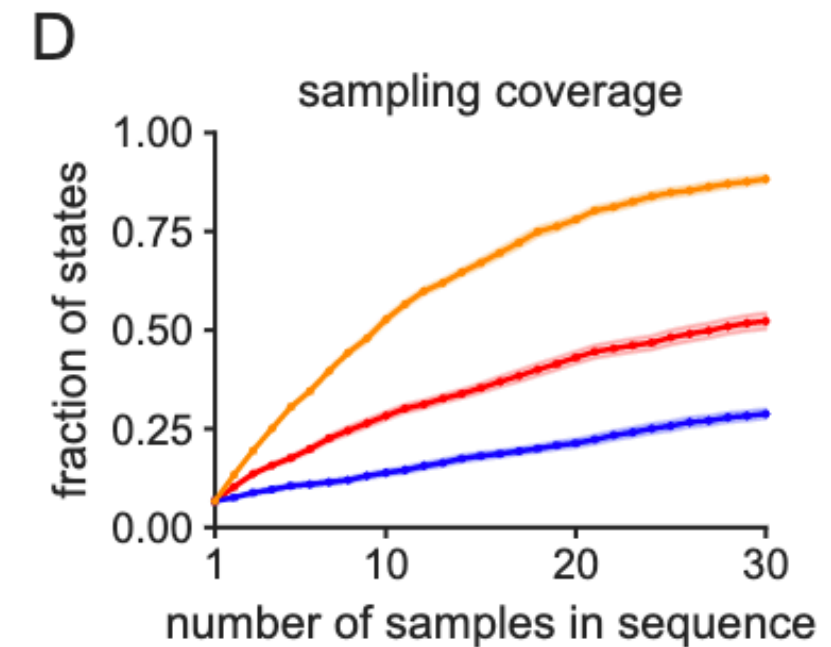
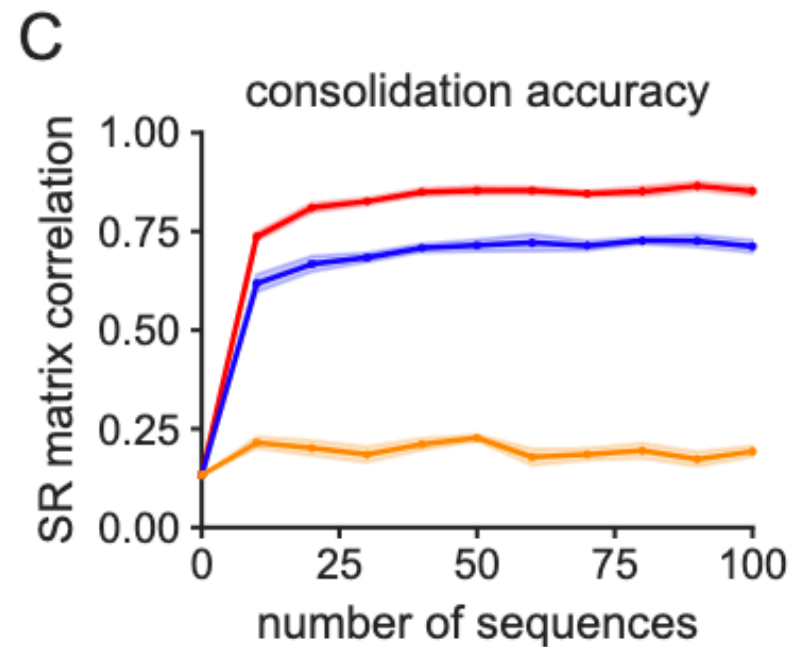
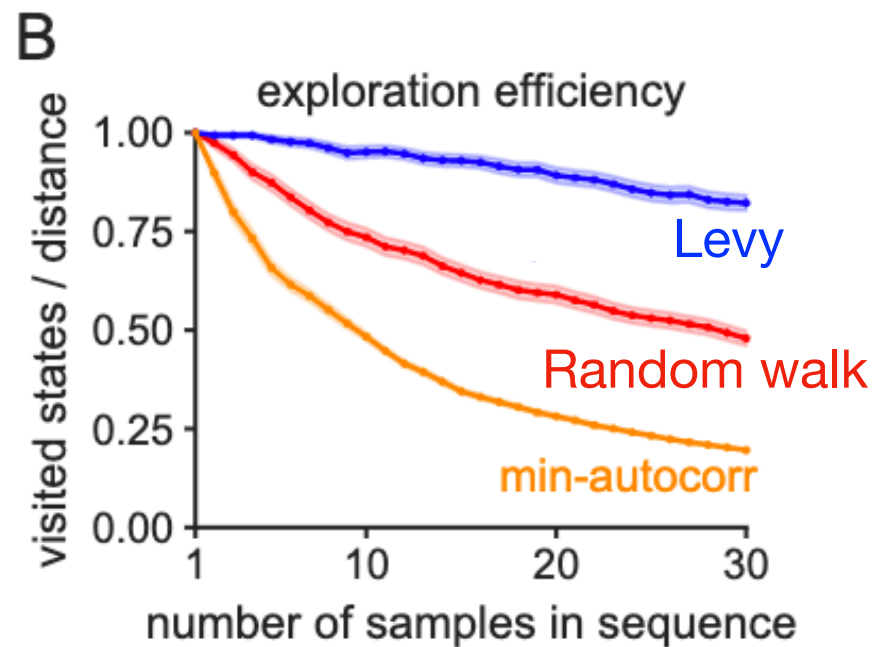
Dan  
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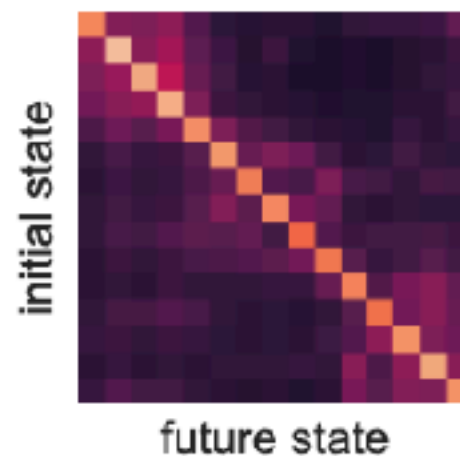


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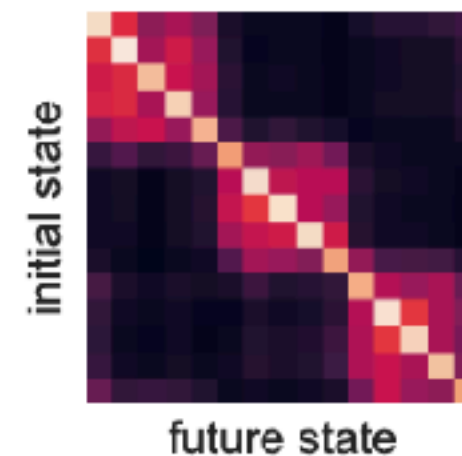
Dan  
McNamee



Levy



Random Walk





# Take-aways

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- Prediction and compression as important ingredients of representations that support flexible behavior
- These are useful terms for understanding the neural mechanisms of cognitive flexibility
- Representation learning is useful for neuroscience and neuroscience-inspired AI
- Hippocampus is central to key cognitive functions we don't know how to implement in machines



# Hippocampus and Neuro-Inspired AI

## Navigation

O'Keefe & Nadel 1978

## Structure Learning

Schapiro et al 2016

## Relational Memory

Eichenbaum et al 1999

## Model-based Planning

Miller et al 2017, Vikhbladh et al 2019

## Rapid, Episodic memory

Scoville & Milner 1957

## Replay + Sampling

Skaggs & McNaughton 1996

## Imagination

Hassabis et al 2007

## Binding problem

Opitz 2010

Lots of potential for bridging Neuro + AI

# Acknowledgements



Dan  
McNamee



Jesse  
Geerts



Matt  
Botvinick



Sam  
Gershman



Tim  
Behrens



Neil  
Burgess



Peter Battaglia



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[www.neurokim.com](http://www.neurokim.com)